99. *Some Results on Additive Number Theory. II*

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In this note we outline the proof of the

Theorem. *Let k be an integer >1, and let $\alpha_i<\beta_i$ $(i=1,\cdots,k)$. For sufficiently large positive integer N, let $A(N)$ denote the number of representations of N as the sum of k positive integers: $N=n_1+\cdots+n_k$ such that*

$$\log\log N+\alpha_i\sqrt{\log\log N}<\omega(n_i)<\log\log N+\beta_i\sqrt{\log\log N}\quad (i=1,\cdots,k)$$

simultaneously, where $\omega(n_i)$ denotes the number of distinct prime factors of n_i. Then, as $N\to\infty$, we have

$$A(N)\sim\frac{N^{k-1}}{(k-1)!}(2\pi)^{-k/2}\prod_{i=1}^{k}\int_{\alpha_i}^{\beta_i}e^{-x^2/2}dx.$$

This theorem was announced as Theorem 3 in [2] without proof. Our proof is elementary and makes no use of any limit theorems in probability theory.

Lemma 1. *Let a_i $(i=1,\cdots,k)$ and b be positive integers such that $d=(a_1,\cdots,a_k)$ divides b. Let S denote the number of solutions of the Diophantine equation $a_1x_1+\cdots+a_kx_k=b$ in positive integers, then we have $|S-db^{k-1}/[(k-1)!\,a_1\cdots a_k]|<Cb^{k-2}$, where C is a positive number dependent only on k and independent of a_i and b.*

We define the set P_N consisting of primes as $P_N=\{p: e^{(\log\log N)^2}<p<N^{(\log\log N)^{-2}}\}$ and put $y(N)=\sum_{p\in P_N}1/p$. Then we have

$$(1)\qquad y(N)=\log\log N+O(\log\log\log N).$$

We denote by $\omega_N(n)$ the number of primes p such that $p|n$, $p\in P_N$.

For any positive integer t, we define the set $M(t)$ consisting of positive integers as $M(t)=M(N;t)=\{m: m \text{ is squarefree}; m \text{ has } t \text{ prime factors}; p|m\Rightarrow p\in P_N\}$. We put for convenience $M(0)=\{1\}$.

For any k positive integers t_i, we denote by $F(N;t_1,\cdots,t_k)$ the number of representations of N as the sum of k positive integers: $N=n_1+\cdots+n_k$ such that $\omega_N(n_i)=t_i$ simultaneously.

For any k positive integers $m_i\in M(t_i)$ with some positive integers t_i, we denote by $G(N;m_1,\cdots,m_k)$ the number of representations of N as the sum of k positive integers: $N=n_1+\cdots+n_k$ such that $\prod_{p|n_i,p\in P_N}p=m_i$ simultaneously. We have

$$F(N;t_1,\cdots,t_k)=\sum_{m_1\in M(t_1)}\cdots\sum_{m_k\in M(t_k)}G(N;m_1,\cdots,m_k).$$