115. A Note on the Classification of Stability

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(Communicated by Kenjiro SHODA, M. J. A., Oct. 12, 1976)

1. Introduction. We shall consider the system of ordinary differential equations $\dot{x}=f(t,x)$. Let R^n denote Euclidean space of dimension n. We shall assume that f is continuous on $[0,\infty)\times R^n$ and satisfies the equality $f(t,0)=0$ for $t\geqq 0$. In this note we discuss various types of stability, that is, (simple) stability (abbreviated by S), uniform stability (US), quasi-asymptotic stability (in the large) ($QAS(L)$), stability (in the large) ($AS(L)$), quasi-equi-asymptotic stability (in the large) ($QEAS(L)$), equi-asymptotic stability (in the large) ($EAS(L)$), quasi-uniform-asymptotic stability (in the large) ($QUAS(L)$), uniform-asymptotic stability (in the large) ($UAS(L)$), and exponential-asymptotic stability (in the large) ($Exp\ AS(L)$) introduced by Lyapunov, Massera and many others. For the definitions of the above notions we shall employ those in Yoshizawa [4].

Our purpose is to clarify the relations between these notions. This note is based on a portion of a dissertation of the author's Master degree in 1975 submitted to Osaka University.

We now define $F(S)$ as the family of continuous functions f for which the trivial solutions $x(t)=0$ of $\dot{x}=f(t,x)$ are stable. Of course, in a way similar to the above notation we also define $F(US), F(QAS), \cdots, F(Exp\ ASL)$ respectively. It is convenient to define

$$F_{Lin}(*)=\{f\in F(*)\mid f(t,x)=A(t)x\},$$
$$F_{Aut}(*)=\{f\in F(*)\mid f \text{ is independent of } t\}$$
and $$F_{Per}(*)=\{f\in F(*)\mid f(t+\omega,x)=f(t,x) \text{ for some } \omega>0\}.$$

The author wishes to express his thanks to Professor M. Yamamoto of Osaka University for his kind advice and constant encouragement.

2. Well-known Relations. First we begin to give the propositions, immediate consequences of the definitions.

Proposition 1. $F(US)\subset F(S)$.

Proposition 2. (i) $F(QUAS)\subset F(QEAS)\subset F(QAS)$,
(ii) $F(Exp\ AS)\subset F(UAS)\subset F(EAS)\subset F(AS)$.

Proposition 3. (i) $F(QUASL)\subset F(QEASL)\subset F(QASL)$,
(ii) $F(Exp\ ASL)\subset F(UASL)\subset F(EASL)\subset F(ASL)$.

Proposition 4. (i) $F(Exp\ ASL)\subset F(Exp\ AS)$,
(ii) $F(QUASL)\subset F(QUAS)$ hence $F(UASL)\subset F(UAS)$,
(iii) $F(QEASL)\subset F(QEAS)$ hence $F(EASL)\subset F(EAS)$,