

## Paper Communicated.

### Algebraic Means.

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(Read December 12, 1917.)

For several years, interrupted only by an illness two years ago which almost proved fatal, my thought has been constantly directed to getting insight into the axiomatic nature of the means or averages of any number of positive quantities. Hereby the word *quantity* is used as synonymous with *number*.

The present communication is fragmentary in substance and somewhat vague in many respects. I crave the indulgence of the reader for this and many other shortcomings which it may not be necessary to enumerate in this place.

#### §. 1.

Gauss in his *Theoria Motus Corporum Cœlestium* (Werke Bd. VII, p. 232) takes it as an axiom, that, if any quantity has been determined by direct observations, the arithmetic mean of all the observed values is its most probable value, and, even if this be not strictly true, it is the nearest approach to the most probable value, so that we may safely accept it as such. Encke's so-called proof that the arithmetic mean is the most probable value for any number of observations (*Berliner Astronomisches Jahrbuch* for 1834, pp. 260–262), though consecrated by lapse of time, is not free of serious objections, and the foundation on which it rests, may ultimately be traced to the principle called *the equal distribution of ignorance* by Boole and *the want of sufficient reason* by De Morgan, for which I venture to suggest the name *no reason for preference*, and which was ably refuted by Johannes von Kries and his followers.