2. *Projective Differential-Geometrical Properties of the One-Parameter Families of Point-Pairs in the One-Dimensional Space.*

By Akitsugu KAWAGUCHI.
Mathematical Institute, Tôhoku Imperial University, Sendai.

(Rec. Dec. 20, 1926. Comm. by M. FUJIWARA, M.I.A., Jan. 12, 1927.)

1. When we consider point-pairs in the one-dimensional space as space element, we can treat a one-parameter family of point-pairs in quite similar way as a curve in the two-dimensional space.

In the homogeneous point-coordinates (x_1, x_2) a point-pair is represented by the equation

$$\sum_{i,k} a_{ik}\, x_i\, x_k = 0 \qquad (i, k = 1, 2),$$

therefore we take a_{ik} as the homogeneous coordinates of a point-pair, and

$$\overset{*}{a}_{ik} = |\, a_{lm}\, |^{-\frac{1}{2}}\, a_{ik}$$

as its *normalized coordinates.*

2. Let $a_{ik}(t)$ be the normalized coordinates of a one-parameter family F of point-pairs, and let

$$(m, n) = \begin{vmatrix} a_{11}^{(m)} & a_{12}^{(n)} \\ a_{21}^{(m)} & a_{22}^{(n)} \end{vmatrix} + \begin{vmatrix} a_{11}^{(n)} & a_{12}^{(m)} \\ a_{21}^{(n)} & a_{22}^{(m)} \end{vmatrix},$$

where

$$a_{ik}^{(m)} = \frac{d^m a_{ik}(t)}{dt^m}.$$

Then evidently the relations

$$(0, 0) = 2, \qquad (1, 0) = 0$$

hold good, while $(1, 1)$ does not identically vanish in general. So as the natural parameter we adopt

$$p = \frac{1}{i\sqrt{2}} \int \sqrt{(1, 1)}\, dt$$

instead of t and denote $(2, 2)$ by $2I$. We call the quantity p the *projective*