98. On the System of Linear Inequalities and Linear Integral Inequality.

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1. Dines and Carver[1] have treated in several papers the condition for the inconsistency of a system of linear inequalities

$$L_i(u)=a_{i1}u_1+a_{i2}u_2+\dots+a_{in}u_n\geqq 0,\ (i=1,2,\dots,m) \quad (1)$$

and Dines[2] has extended their investigation to the problem of linear integral inequality. On the other hand Kakeya[3] has treated, as an application of his theory on the system of linear integral equations, the problem of linear differential inequality. It is, however, not remarked by any one that these three problems belong to the same category.

2. Taking this fact into account we can solve the problem of linear inequality in the following manner.

First consider the system of non-homogeneous linear equations

$$L_i(u)=b_i,\ (i=1,2,\dots,m) \quad (2)$$

and its adjoint system

$$M_k(v)=a_{1k}v_1+a_{2k}v_2+\dots+a_{mk}v_m=0. \quad (3)$$

Then there exists the relation

$$\sum_i v_iL_i(u)-\sum_k u_kM_k(v)=0. \quad (4)$$

Let $\qquad v_1^{(\lambda)},\ v_2^{(\lambda)},\ \dots,\ v_m^{(\lambda)},\ (\lambda=1,2,\dots,s)$

be linearly independent solutions of (3). ($s=m-r$ if r be the rank of the matrix (a_{ik})).

It will easily be proved that (2) has solution when and only when $(b_1, b_2,\dots, b_m)$ satisfy the condition

$$\sum_i b_i v_i^{(\lambda)}=0,\ \lambda=1,2,\dots,s. \quad (5)$$

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1) Dines, Annals of Math , (2) **20** (1918–19), **27** (1925–26), **28** (1928), p. 41, 386. Carver, ibid. (2) **23** (1923).

2) Dines, Trans. American Math. Society, **30** (1928), Annals of Math., (2) **28** 1926–27), p. 393.

3) Kakeya, Proc. Math.-Phy. Soc. Japan. (2) **8** (1915). See also Fujiwara, Science Reports, Tohoku University, **4** (1915).