

PAPERS COMMUNICATED

87. Über die Fermatsche Vermutung, III.

Von Taro MORISHIMA.

Shizuoka Kotogakko.

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Satz: Es sei

$$x^p + y^p + z^p = 0, \quad (x, y, z) = 1, \quad xyz \not\equiv 0 \pmod{p}, \quad p > 3.$$

Dann ist

$$\sum_{i=1}^{p-1} \frac{t^i}{i} \equiv 0 \pmod{p^2},$$

$$\text{wo} \quad t \equiv -\frac{y}{x}, \quad -\frac{x}{y}, \quad -\frac{z}{y}, \quad -\frac{y}{z}, \quad -\frac{x}{z}, \quad -\frac{z}{x} \pmod{p^2}.$$

Beweis: Es ist

$$\left[\sum_{i=1}^{p-1} \frac{t^i}{i} \right]^2 = 2 \left[\sum_{i=2}^p \frac{t^i}{i} \sigma_i + \sum_{i=1}^{p-2} \frac{t^{p+i}}{p+i} (\sigma_p - \sigma_{i+1}) \right],$$

$$\text{wo} \quad \sigma_i = 1 + \frac{1}{2} + \dots + \frac{1}{i-1}, \quad \text{also wegen } \sigma_p \equiv 0 \pmod{p^2},$$

$$\begin{aligned} \left[\sum_{i=1}^{p-1} \frac{t^i}{i} \right]^2 &\equiv 2 \left[\sum_{i=2}^{p-1} \frac{t^i}{i} \sigma_i - t \sum_{i=1}^{p-2} \frac{t^i}{i} \sigma_{i+1} \right] \\ &\equiv 2 \left[\sum_{i=2}^{p-1} \frac{t^i}{i} \sigma_i - t \sum_{i=2}^{p-1} \frac{t^i}{i} \sigma_i - t \sum_{i=1}^{p-1} \frac{t^i}{i^2} \right] \pmod{p}, \end{aligned}$$

$$\text{also wegen}^* \quad \sum_{i=1}^{p-1} \frac{t^i}{i^e} \equiv 0, \quad 1 \not\equiv t \pmod{p}, \quad e=1, 2$$

$$\sum_{i=2}^{p-1} \frac{t^i}{i} \sigma_i \equiv 0 \pmod{p}. \quad (1)$$

Nun ist

$$(1-t)^p \equiv 1 - t^p \pmod{p^3},$$

* Das Kummersche Kriterium.