14. *A Theorem concerning the Dynamical Systems with Slow Variation.*

By Yusuke HAGIHARA.
Astronomical Observatory, Azabu, Tokyo.
(Comm. by K. HIRAYAMA, M.I.A., Feb. 12, 1931.)

1. In certain dynamical systems such as often appear in celestial mechanics the solution is not always convergent[1] if it is expressed in series of a form for practical use. Although its convergence in series of a certain form is actually proved in the way of the theory of analytic functions,[2] the solution in convergent series can not reveal its nature from the dynamical point of view. Thus we have to employ *semi-convergent* series in order to get the information as to the behaviour of the motion. It is proposed to estimate the duration of time in which the solution can be approximated by an adopted representation to a previously given degree. A first step toward attacking this problem was trodden by Birkhoff[3] for a motion near an equilibrium point in which the characteristic numbers are all distinct, different from zero and not connected by any linear homogeneous relation with rational coefficients. The problem of this note[4] is to extend the research to the case in which a certain number of characteristic numbers are zero at least in the first approximation.

2. Consider a function H of $2m+2n$ variables x_i, y_i, ξ_j, η_j and t $(i=1, 2, \ldots\ldots, m;\ j=1, 2, \ldots\ldots, n)$, which, together with its partial derivatives of the first order with respect to any of the variables, satisfy the Lipschitz condition for all the variables ξ_j, η_j and t and for all values of ξ_j, η_j and t in a domain: $|x_i|$, $|y_i|<D$, $(i=1, 2, \ldots\ldots, m)$ with a finite positive constant D. With this function H associate the following system of differential equations:

---

1) H. Poincaré: Acta Math. **13** (1889), 1; *Méthodes Nouvelles de la Mécanique Céleste*. T. **2** (1893).

2) K. Sundman: Acta Math. **36** (1912), 105; T. Levi-Civita: Acta Math. **42** (1918), 99.

3) G. D. Birkhoff: Amer. Jour. Math. **49** (1927), 1; *Dynamical Systems*. (1927) Chap. IV.

4) It is a great pleasure to express my indebtedness to Professor G. D. Birkhoff for his valuable criticisms and suggestions and to the Rockfellor Foundation for enabling me to work at the Harvard University.