133. *On the Theory of Schlicht Functions.*

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1. Let the function

(1) $$f(z)=z+a_2z^2+\cdots\cdots$$

be regular and schlicht in $|z|<1$. It is well known that $|a_n|<en$. Recently Littlewood and Paley[1] show that the coefficients a_n are uniformly bounded, if $f(z)$ be odd. We can, however, proceed one step further, and prove that

(2) $$|a_n|<e^2,$$

for the odd schlicht function.

If the function (1) satisfies the relation

(3) $$f(\omega z)=\omega f(z),$$

where ω denotes any root of the equation $x^k=1$, k being a positive integer, we say, for convenience, that $f(z)$ is a function of the class k. Using this definition, the results stated above may be expressed as follows: if $f(z)$ is a function of the class k, then, for $k=1, 2$, we have

(4) $$n^{\frac{k-2}{k}}|a_n|<A.$$

The question arises whether the inequality (4) holds good also for $k>2$. The answer is affirmative in the case $k=3$, and the relation

(5) $$\sqrt[3]{n}\,|a_n|<e^3$$

can be demonstrated. Thus we can state the following theorem:

If $f(z)=z+\sum_2^\infty a_nz^n$ *is a function of the class* k ($k=1, 2, 3$), *then*

(6) $$n^{\frac{k-2}{k}}|a_n|<e^k.$$

In general, if the function $zf'(z)$ is p-valent, i.e. the equation

$$zf'(z)=a$$

has at most p solutions in $|z|<1$, for every a, we can establish the inequality

(7) $$n^{\frac{k-2}{k}}|a_n|<p\,e^{\frac{1}{k}},$$

1) J. E. Littlewood and R. E. A. C. Paley: Journal of London Math. Soc. **7** (1933). See also E. Landau: Über ungerade schlichte Funktionen, Math. Zeits. **37** (1933).