# 162. On the Summability of Fourier Series by Riesz's Logarithmic Means.

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1. Let $f(t)$ be a summable and periodic function with period $2\pi$, and let

$$f(t) \sim \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos nt + b_n \sin nt)\,. \qquad (1\cdot 1)$$

The Fourier series $(1\cdot 1)$ is said to be summable $(R, k)$, for $t=x$, to sum $s$, provided that

$$R_\omega^k = \frac{a_0}{2} + \frac{1}{(\log\omega)^k}\sum_{n<\omega}\left(\log\frac{\omega}{n}\right)^k (a_n \cos nx + b_n \sin nx)$$

tends to a limit $s$, as $\omega \to \infty$.[1]

Let $$\phi(u) = \frac{1}{2}\{f(x+u) + f(x-u) - 2s\}\,;$$

we write $$\phi(t) \to 0 \quad (R, \alpha)$$

as $t \to 0$, provided that

$$\phi_\alpha(t) = \frac{1}{\Gamma(\alpha)}\int_t^\pi \left(\log\frac{u}{t}\right)^{\alpha-1}\frac{\phi(u)}{u}\,du = o\left[\left(\log\frac{1}{t}\right)^\alpha\right],$$

when $t \to 0$.

Concerning the summability of Fourier series by Riesz's logarithmic means, Prof. Hardy has given a theorem on $(R, 1)$ summability.[2] Now we can extend this theorem and obtain some other theorems. The proof of them will appear in Tohoku Mathematical Journal.

2. Suppose that $k$ is a positive integer and $\phi_0(t) = \phi(t)$, then we have

Theorem A. *If*

$$\int_0^t |\phi_{k-1}(u)|\,du = O\left[t\left(\log\frac{1}{t}\right)^k\right],$$

*then the necessary and sufficient condition that the series* $(1\cdot 1)$ *should be summable* $(R, k)$, *for* $t=x$, *to sum* $s$, *is that*

1) Hardy and Riesz: Theory of general Dirichlet's series.
2) Hardy: Quarterly Journal, **2** (1931).