103. *Notes on Fourier Series (II): Convergence Factor.*

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1. R. Salem has proved the following theorem:[1]

If $f(x)$ is a continuous function with period 2π and its Fourier coefficients be a_n and b_n, then the relation

$$\text{(1)} \qquad \lim_{s\to 0}\left\{\frac{1}{2}a_0+\sum_{n=1}^{\infty}\frac{a_n\cos nx+b_n\sin nx}{1+s\log n}\right\}=f(x)$$

holds good almost everywhere.

In this relation we must notice that the series in the bracket of the right hand side is convergent for every positive value of s and for almost all x.

One of the present authors[2] has proved that

$$\text{(2)} \qquad \lim_{s\to 0}\left\{\frac{1}{2}a_0+\sum_{n=1}^{\infty}\frac{a_n\cos nx+b_n\sin nx}{1+s\sqrt{\log n}}\right\}=f(x)$$

almost everywhere for squarely integrable function $f(x)$.

The object of this paper is to prove that (1) is true for any integrable function and there is the corresponding relation for the function in L^p $(1\leqq p\leqq 2)$.

2. *Theorem. If $f(x)\varepsilon L^p(1\leqq p\leqq 2)$ and is periodic with period 2π and a_n and b_n are its Fourier coefficients, then we have*

$$\text{(3)} \qquad \lim_{s\to 0}\left\{\frac{1}{2}a_0+\sum_{n=1}^{\infty}\frac{a_n\cos nx+b_n\sin nx}{1+s(\log n)^{1/p}}\right\}=f(x)$$

almost everywhere.

Actually we can replace the factors $\left\{\dfrac{1}{1+s(\log n)^{1/p}}\right\}$ by the more general sequence $\{\psi_n(s)\}$ which satisfies certain conditions.[3] But we make here no attention to this.

For the proof we make use of the theorem:

Lemma. If $f(x)\varepsilon L^p$ $(1\leqq p\leqq 2)$, then

$$\sum_{n=1}^{\infty}\frac{a_n\cos nx+b_n\sin nx}{(\log n)^{1/p}}$$

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1) R. Salem, Comptes Rendus, **205** (1937), pp. 14–16, **205** (1937), pp. 311–313. In the latter paper, Salem remarked that more generally for a bounded function, (2) holds good.

2) T. Kawata, Proc. **13** (1937), 381–384.

3) Cf. Salem, loc. cit. and T. Kawata, loc. cit.