PAPERS COMMUNICATED

28. A Problem of Diophantine Approximations in the Old Japanese Mathematics.

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In studying the history of the old Japanese mathematics, so-called Wazan, I have found that a manuscript with the title *Ruiyaku-zitu* (累約術), or the method of successive divisions, written by *Katahiro Takebe* (建部賢弘, 1664-1739), revised in 1728 by his pupil, *Genkei Nakane* (中根元圭, 1662-1733), contains problems of Diophantine approximations. This manuscript has been mentioned by many mathematicians in our country, but it seems that the importance of its content was not sufficiently perceived by them.

This manuscript consists of the following three problems. The first treats of the integral solutions of $|ax-by| < 1$, while the second and the third those of $|ax-by+c| < 1$ and $|ax-by-c| < 1$ respectively, where a, b, c are given positive real numbers.

The author of this manuscript solved the first problem by expanding b/a into simple continued fractions, quite similar to the modern theory of continued fractions.

For the second and the third problems *Takebe* developed an algorithm very similar to the *Jacobi algorithm* and gave the concrete form for the solutions, which is very remarkable.

I will translate freely the second problem in the following lines.

Problem. Let $c = 5513.9106$, the initial additive number (原益數), be added repeatedly by the successive additive number (累益數) $a = 954.5338$ and subtracted repeatedly by the successive subtractive number (累損數) $b = 6034.4574$. What are the integral values of x, y, which are called the additive multiplier (益段數) and the subtractive multiplier (損段數), such that $ax-by+c$ lies between two given limits (許限) $-\delta$ and $+\delta$? [Here it is assumed $b > c$, $\delta = 1$].[1]

The solutions x, y of $0 < ax-by+c < 1$ are called the strong additive and subtractive multipliers (強益段, 強損段), while the solutions x, y of $-1 < ax-by+c < 0$ the weak additive and subtractive multipliers (弱益段, 弱損段).

Answer: the strong additive multiplier   15034,
　　　　the strong subtractive multiplier 2379,
　　　　the weak additive multiplier   854,
　　　　the weak subtractive multiplier   136.

Solutions : Since there is the initial additive number c, we solve this problem by two processes.

The first process runs similarly as the first problem. Divide b by a and let the quotient (商) be a_1, the remainder (不盡) be r_1. Next

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1) [　] is the remark of the author of this paper.