35. Operator-theoretical Treatment of Markoff's Process, II.

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(Comm. by T. TAKAGI, M.I.A., May 12, 1939.)

§ 1. Let $P(x, E)$ denote the transition probability that the point x of the interval $\Omega=(0,1)$ is transferred, by a simple Markoff's process, into the Borel set E of Ω after the elapse of a unit time. It is naturally assumed that $P(x, E)$ is completely additive for Borel sets E if x is fixed and that $P(x, E)$ is Borel measurable in x if E is fixed. $P(x, E)$ defines a linear operator P on the complex Banach space $(\mathfrak{M})$ in $(\mathfrak{M})$[1]:

$$P\cdot f=g\,,\quad g(E)=\int_\Omega P(x, E)f(dx)\,.$$

It is easy to see that the iterated operator P^n is defined by the kernel $P^{(n)}(x, E)=\int_\Omega P^{(n-1)}(x, dy)\,P(y, E)\,\bigl(P^{(1)}(x, E)=P(x, E)\bigr)$. In the preceding note,[2] it is proved that the following condition (D) implies the condition (K):

(D) $\begin{cases}\text{there exist an integer } s \text{ and positive constants } b, \eta\ (<1) \text{ such} \\ \text{that, if mes } (E)<\eta,\ P^{(s)}(x, E)<1-b \text{ uniformly in } x, E.\end{cases}$

(K) $\begin{cases}\text{there exist an integer } n \text{ and a completely continuous linear} \\ \text{operator } V \text{ such that } \| P^n-V\|_{\mathfrak{M}}<1.\end{cases}$

The condition (K) is more general than (D), since there exists $P(x, E)$ which satisfies (K) but not (D). In [I] it is proved that, if $P(x, E)$ satisfies (D), then

(B) $\begin{cases}\text{the proper values } \lambda \text{ with modulus 1 of } P \text{ are all roots of} \\ \text{unity.}\end{cases}$

Thus, combined with (K), we were able to give an operator-theoretical treatment of the Markoff's process $P(x, E)$ under the condition (D). (See [I].)

In the present note I intend to show that the condition (K) im-

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1) $(\mathfrak{M})$ is the linear space of all the totally additive set functions defined for all the Borel sets of Ω. For any $f\in(\mathfrak{M})$ we define its norm $\|f\|_{\mathfrak{M}}$ by the total variation of f on Ω.

2) K. Yosida: Operator-theoretical Treatment of the Markoff's Process, Proc. **14** (1938), 363. This note will be referred to as [I] below. It contains many misprints. On page 364, line 27 and line 28 $(\mathfrak{M})$ is to be read (M^*). On page 364, line 28 $h(dz)$ is to be read $h(z)$. On page 365, line 7 "$f_{i_k}(E)\cdot f_{j_k}(E)\equiv 0$ for $i\neq j$" is to be read "$f_{i_k}(E_{i_k})=1$ where $E_{i_k}\cdot E_{j_k}=$void for $i\neq j$." On page 367, line 4 and 5 "From... by (6)" is to be read "Evident from iii) and the equations $f_{(i+1)_a}(E)=\int_0^1 P(x, E)f_{i_a}(dx)$ below."