38. *On Affine Geometry of Abelian Groups.*

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1. Let a set G of elements $a, b, c, \dots$, satisfy the following axioms:

(1) For any pair of elements a, b, the product $a\cdot b$ of G determines uniquely one element c in G, viz. $a\cdot b=c$.

(2) For two given elements a (or b) and c, the equation $a\cdot b=c$ can be uniquely solved by b (or a) in G.

(3) For any four elements a, b, c and d, we have

$$(a\cdot b)(c\cdot d)=(a\cdot c)\cdot(b\cdot d).$$

(4) Each element a in G is idempotent, viz. $a\cdot a=a$.

In my previous paper[1], we know that the operation $a\cdot b$ represents an abstract generalization of the mean operation which divides the straight line joining two points a, b in a given ratio $m:n$.

Now, we apply

Definition (A). *Let us suppose that a set $L(a, b)$ consists of elements which are produced from two elements a, b by the operation (1) and its inverse operation (2) with possible repetitions. Then, we say that the set $L(a, b)$ is the straight line*[2] *joining two points a, b in the space G.*

Under the above definition (A), can we constitute a space of affine geometry from G?[3] But the answer for this problem is not always affirmative, because the straight line $L(a, b)$ of G does not necessarily admit the familiar proposition:

(L) *Any two straight lines, not parallel to each other, meet in one and only one point.*

In the following lines, we shall proceed to find a necessary and sufficient condition of the problem.

2. In place of the product $a\cdot b$, let us introduce the new product $a+b$ into G as follows:

(5) Let a and b be any two given elements in G. If $a=x\cdot s$ and $b=s\cdot y$, for a fixed element s, then we put $a+b=x\cdot y$.

Here, we know[4] that the set G forms an abelian group with respect to the new product $a+b$ and moreover the old product $a\cdot b$

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1) K. Toyoda, On Axioms of Mean Transformation and Automorphic Transformations of Abelian Groups, Tôhoku Math. Journal **47** (1940).

2) This definition is due to the remark of M. Takasaki.

3) G. Hessenberg, Acta Math. 29;
H. Wiener, Jahrsber. d. D. M. V. (1891);
G. Hassenberg, Grundlagen der Geometrie;
M. Pasch und M. Dehn, Vorlesungen über neue Geometrie;
K. Reidmeister, Vorlesungen über Grundlagen der Geometrie;
O. Veblen and J. W. Young, Projective Geometry, I, II;
H. Weyl, Raum, Zeit, Matrie.

4) K. Toyoda, loc. cit.