89. Note on the Kronecker Product of Representations of a Group.

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The principal aim of this note is to prove the following Theorem 1. As an application we can prove a conjecture of R. Brauer-C. Nesbitt[1] (Th. 5).

Let $\mathfrak{G}$ be a group of finite order. We consider representations of $\mathfrak{G}$ in an arbitrary field K.

Theorem 1. Let $\mathfrak{G}$ be a group of finite order and R be its regular representation. If V is a representation of $\mathfrak{G}$ of degree m, then

$$V\times R\cong\begin{pmatrix} R & & 0 \\ & R & \\ & & \ddots \\ 0 & & & R \end{pmatrix}$$

where R appears m times.

Proof[2]. We denote by $G_1, G_2, \dots, G_t$ the elements of $\mathfrak{G}$. Let G be an element of $\mathfrak{G}$. If $GG_i=G_j$, then

$$R(G)=\begin{array}{cc} & \quad i \\ j & \begin{pmatrix} & 0 & \\ * & \vdots & * \\ 0\cdots & 1 & \cdot\cdot 0 \\ * & \cdot & * \\ & 0 & \end{pmatrix}\end{array}$$

and

$$V(G)\times R(G)=\begin{pmatrix} & 0 & \\ * & \vdots & * \\ 0\cdot\cdot & V(G) & \cdot\ 0 \\ * & \cdot & * \\ & 0 & \end{pmatrix}.$$

If we put

$$P=\begin{pmatrix} V(G_1) & & 0 \\ & V(G_2) & \\ & & \ddots \\ 0 & & & V(G_t) \end{pmatrix}$$

then it follows from $GG_i=G_j$ that

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1) R. Brauer-C. Nesbitt, On the modular characters of groups, Ann. of Math. **42** (1941), p. 579.

2) If R is completely reducible, we can easily see the validity of this theorem by comparing the characters of the representations.