On the Structure of Painlevé Transcendents with a Large Parameter. II.

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§0. Introduction. The purpose of this note is to report a result on the structure of 2-parameter formal solutions of the Painlevé equations with a large parameter η, which are tabulated in Table 0.1 below. The formal solutions to be considered here have been constructed in [1] by the so-called multiple-scale analysis, and the main result (Theorem 2.1) of this note asserts that any of them can be locally reduced to a 2-parameter formal solution of the first Painlevé equation (P_{I}); this is a natural generalization of the result on 0-parameter solutions reported in our precedent note [3]. (See [4] for the details of the proof of the results announced in [3].)

Table 0.1. Painlevé equations with a large parameter η.

$$(P_{\mathrm{I}}) \quad \frac{d^2\lambda}{dt^2} = \eta^2(6\lambda^2 + t).$$

$$(P_{\mathrm{II}}) \quad \frac{d^2\lambda}{dt^2} = \eta^2(2\lambda^3 + t\lambda + \alpha).$$

$$(P_{\mathrm{III}}) \quad \frac{d^2\lambda}{dt^2} = \frac{1}{\lambda}\left(\frac{d\lambda}{dt}\right)^2 - \frac{1}{t}\frac{d\lambda}{dt} + 8\eta^2\left[2\alpha_\infty\lambda^3 + \frac{\alpha'_\infty}{t}\lambda^2 - \frac{\alpha'_0}{t} - 2\frac{\alpha_0}{\lambda}\right].$$

$$(P_{\mathrm{IV}}) \quad \frac{d^2\lambda}{dt^2} = \frac{1}{2\lambda}\left(\frac{d\lambda}{dt}\right)^2 - \frac{2}{\lambda} + 2\eta^2\left[\frac{3}{4}\lambda^3 + 2t\lambda^2 + (t^2 + 4\alpha_1)\lambda - \frac{4\alpha_0}{\lambda}\right].$$

$$(P_{\mathrm{V}}) \quad \frac{d^2\lambda}{dt^2} = \left(\frac{1}{2\lambda} + \frac{1}{\lambda - 1}\right)\left(\frac{d\lambda}{dt}\right)^2 - \frac{1}{t}\frac{d\lambda}{dt} + \frac{(\lambda-1)^2}{t^2}\left(2\lambda - \frac{1}{2\lambda}\right) + \eta^2\frac{2\lambda(\lambda-1)^2}{t^2}\left[(\alpha_0 + \alpha_\infty) - \alpha_0\frac{1}{\lambda^2} - \alpha_2\frac{t}{(\lambda-1)^2} - \alpha_1 t^2\frac{\lambda+1}{(\lambda-1)^3}\right].$$

$$(P_{\mathrm{VI}}) \quad \frac{d^2\lambda}{dt^2} = \frac{1}{2}\left(\frac{1}{\lambda} + \frac{1}{\lambda-1} + \frac{1}{\lambda-t}\right)\left(\frac{d\lambda}{dt}\right)^2 - \left(\frac{1}{t} + \frac{1}{t-1} + \frac{1}{\lambda-t}\right)\frac{d\lambda}{dt} + \frac{2\lambda(\lambda-1)(\lambda-t)}{t^2(t-1)^2}\left[1 - \frac{\lambda^2 - 2t\lambda + t}{4\lambda^2(\lambda-1)^2} + \eta^2\left\{(\alpha_0 + \alpha_1 + \alpha_t + \alpha_\infty) - \alpha_0\frac{t}{\lambda^2} + \alpha_1\frac{t-1}{(\lambda-1)^2} - \alpha_t\frac{t(t-1)}{(\lambda-t)^2}\right\}\right].$$

The details of this note shall be published elsewhere. We sincerely thank Professor T. Aoki for the stimulating discussions with him on the subjects discussed here.

§1. A canonical Schrödinger equation (Can) near the double turning point and its isomonodromic deformation. In this note we use the same notions and notations as in [3] except that the formal solution $\lambda_J (J = \mathrm{I, II}, \cdots \mathrm{VI})$ of (P_J) considered in [3] and [4] is denoted by $\lambda_J^{(0)}$ here; in particular (SL_J) denotes the Schrödinger equation tabulated in Table 1.2 of [4], K_J denotes the Hamiltonian tabulated in Table 1.3 of [4], and $S_{J,\mathrm{odd}}$ denotes the odd part of a solution S_J of the Riccati equation

$$(1.1) \qquad S_J^2 + \frac{\partial S_J}{\partial x} = \eta^2 Q_J$$

associated with (SL_J). (Cf. [1], Definition 2.1.) In order to save space we also refer the reader to [4] for the definition of the coefficient A_J of the deformation equation (D_J) for (SL_J), i.e.,

$$(D_J) \qquad \frac{\partial \psi}{\partial t} = A_J\frac{\partial \psi}{\partial t} - \frac{1}{2}\frac{\partial A_J}{\partial x}\psi.$$

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