Finding Singular Support of a Function from its Tomographic Data

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Abstract: We develop three different approaches to the problem of finding discontinuity surface S of a function $f(x)$ from its tomographic data: 1) pseudolocal tomography, introduced by the authors, produces S and the jumps of f across S from pseudolocal tomographic data; 2) local tomography, introduced in the literature and further generalized by one of the authors produces S from local data. A method is proposed for finding jumps of f across S. This method is based on the analysis of pseudodifferential operators acting on piecewise smooth functions; and 3) geometric tomography, introduced by one of the authors and A. Zaslavsky, produces S from $\hat{S}$, a dual variety with respect to S, which happens to be the singular support of $\hat{f}(\theta, p)$.

I. Introduction. Let $f(x)$ be a compactly supported piecewise-smooth function. For simplicity we assume that $x \in \boldsymbol{R}^2$, but the basic ideas and results are valid in $\boldsymbol{R}^n$, $n \geq 2$. Define the Radon transform $f(\theta, p) := \int_{\boldsymbol{R}^2} f(x)\delta(p - \Theta \cdot x)dx$, where δ is the delta-function, $\Theta \in S^1$, S^1 is the unit sphere in $\boldsymbol{R}^2$, $\Theta \cdot x$ is the dot product, $p \in \boldsymbol{R}^1$. Here and everywhere below we use the scalar variable θ, $0 \leq \theta < 2\pi$, along with the vector variable Θ so that $\Theta = (\cos\theta, \sin\theta)$. The Radon transform has been studied in [2, 4, 14]. In many applications one is interested in finding discontinuity surfaces of $f(x)$ given $\hat{f}(\theta, p)$. The traditional way to do it is to invert the Radon transform. This is called the standard tomography. The inversion formula is well known [14]. This formula requires integration with respect to θ and p, and therefore is an expensive operation. One is interested in finding a fast and less computationally expensive procedure for finding discontinuity surfaces S of f.

We describe three different approaches to this problem: 1) pseudolocal tomography (PLT), introduced by the authors in [6] and further developed in [13], produces the discontinuity surfaces S of f and jumps of f across S from pseudolocal tomographic data; 2) local tomography (LT), introduced in [31] and [30] and further generalized in [17], produces S from local data, but no methods for finding jumps of f across S were given in the literature. We develop a new method for finding these jumps within the framework of local tomography. Also, we give an approach to optimizing the LT formulas [22]; and 3) geometric tomography, introduced in [24], produces S from $\hat{S}$, a dual variety with respect to S, which happens to be the singular support of $\hat{f}(\theta, p)$.

A fast algorithm to recover S is given by local tomography [31, 1]. This procedure is to calculate $(-\Delta)^{1/2}f$. From ellipticity of $-\Delta$ it follows [3] that $(-\Delta)^{1/2}f$ and f have the same singular support, and one can prove that calculation of $(-\Delta)^{1/2}f$ at a point x given $\hat{f}(\theta, p)$ can be done using only the intergrals of f along straight lines passing through the point x. This results in an inversion formula which uses integration with respect to θ-variable only. Local tomography, as developed in [31] and [1], produces the image of the discontinuity surface S of f, but it does not give the values of jumps of f across S. These jumps are of practical importance in many applications.

In this announcement we present the following new results: 1) we introduce the concept of pseudolocal tomography, construct a formula which allows one to calculate fast the discontinuity curve S of f given the PLT data. By the PLT data we mean the knowledge of the integrals

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