A Table of Absolute Norms of Heilbronn Sums

By Ken YAMAMURA

Department of Mathematics, National Defence Academy

(Communicated by Shokichi IYANAGA, M. J. A., Jan. 12, 1995)

Let p be an odd prime number and ζ a primitive p^2th root of unity. Let L be the unique subfield of $\boldsymbol{Q}(\zeta)$ of degree p. The pth Heilbronn sum is defined as the trace of ζ from $\boldsymbol{Q}(\zeta)$ to L. We denote by NH_p its absolute norm. Fouché[1] proved that if l is a prime divisor of NH_p, then l satisfies the congruence

$$l^{p-1} \equiv 1 \pmod{p^2}.$$

This congruence is well known for $l = 2$, because Wieferich [4] proved that if there exists a counterexample to the first case of Fermat's last theorem for the exponent p (FLT,I,p), then $l = 2$ satisfies the congruence above. Wieferich's result has been generalized as follows [3]: if there exists a counterexample to (FLT,I,p), then all prime numbers l with $2 \leqq l \leqq 113$ satisfy the congruence above. In [1], the table of the values of NH_p for $p < 50$ was given (by the referee). We extend it to $p < 100$ and give complete factorizations of NH_p. The computation was done on a NeXT computer.

Table

p	NH_p (factorization)
3	-1
5	-1
7	97 (prime)
11	$-243 = -3^5$
13	$12167 = 23^3$
17	577 (prime)
19	221874931 (prime)
23	157112485811 (prime)
29	$-2480435158303 = -137 \cdot 18105366119$
31	$310695313260929 = 3727 \cdot 34367 \cdot 2425681$
37	$-5114055181947668782 9$ (prime)
41	$2727257042363914863401 = 17435281 \cdot 156421742922521$
43	$-2572343484535669027372727 = -19^4 \cdot 1973851861584601888 7$
47	$1052824394331287344099620777449 = 53^2 \cdot 374803985165997630508942961$