16. Singular Variation of Domains and L^{∞} Boundedness of Eigenfunctions for some Semi-linear Elliptic Equations

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1. Introduction. Let M be a bounded domain in $\boldsymbol{R}^2$ with smooth boundary ∂M. Let w be a fixed point in M. By $B(\varepsilon\,;w)$ we denote the ball of center w with radius $\varepsilon > 0$. We remove $\overline{B(\varepsilon\,;w)}$ from M and we put $M_\varepsilon = M\setminus\overline{B(\varepsilon\,;w)}$. We write $B(\varepsilon\,;w) = B_\varepsilon$.

Fix $p \in (1,\infty)$. We put

$$\lambda(\varepsilon) = \inf_{X_\varepsilon}\int_{M_\varepsilon} |\nabla u|^2\,dx, \tag{1.1}$$

where $X_\varepsilon = \{u \in H^1(M_\varepsilon) : \|u\|_{L^{p+1}(M_\varepsilon)} = 1,\ u = 0 \text{ on } \partial M,\ u \geq 0 \text{ in } M_\varepsilon\}$. Then, we know that there exists at least one solution u_ε which attains (1.1). It satisfies

$$\begin{aligned} -\Delta u_\varepsilon &= \lambda(\varepsilon)u_\varepsilon^p && \text{in } M_\varepsilon, \\ \frac{\partial}{\partial\nu_x}u_\varepsilon &= 0 && \text{on } \partial B_\varepsilon, \\ u_\varepsilon &= 0 && \text{on } \partial M. \end{aligned} \tag{1.2}$$

Here $\partial/\partial\nu_X$ denotes the exterior normal derivative.

In this paper we prove the following Theorem 1.

Theorem 1. *There exists a positive constant C independent of ε such that*

$$\sup_{u_\varepsilon\in S_\varepsilon}\sup_{x\in M_\varepsilon} u_\varepsilon(x) < C, \tag{1.3}$$

where S_ε is the set of minimizers of (1.1).

The reader may be referred to Ozawa [2],[3], Lin [1] for related problems.

2. Preliminary lemma. Lemma 2.1. *Assume that $u_\varepsilon \in C^\infty(M_\varepsilon)$ is harmonic in M_ε and $u_\varepsilon = 0$ for any $x \in \partial M$ and that*

$$\max\{|\partial u_\varepsilon(x)/\partial\nu_x|\,;x \in \partial B(\varepsilon\,;w)\} = L.$$

Then, $|u_\varepsilon(x)| \leq C\varepsilon L(1+\log(|x-w|/\varepsilon))$ for any $x \in M_\varepsilon$. Here C is a positive constant independent of ε.

Lemma 2.1 is given in Ozawa [4].

Let $G_\varepsilon(x,y)$ be the Green function of the Laplacian in M_ε satisfying

$$\begin{aligned} -\Delta_x G_\varepsilon(x,y) &= \delta(x-y) && x, y \in M_\varepsilon, \\ G_\varepsilon(x,y)_{|x\in\partial M} &= 0 && y \in M_\varepsilon, \\ \frac{\partial}{\partial\nu_x}G_\varepsilon(x,y)_{|x\in\partial B_\varepsilon} &= 0 && y \in M_\varepsilon. \end{aligned}$$

Let $G(x,y)$ be the Green function of the Laplacian in M under the Dirichlet condition on ∂M. We put

$$\langle\nabla_w a(x,w),\nabla_w b(w,y)\rangle = \sum_{i=1}^{2}\frac{2}{\partial w_i}a(x,w)\frac{\partial}{\partial w_i}b(w,y).$$