59. The Explicit Formula for the Harish-Chandra C-function of SU(n, 1) for Arbitrary Irreducible Representations of K which Contain One Dimensional M-types

By Masaaki EGUCHI*) and Shin KOIZUMI**)

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§0. Introduction. The purpose of this note is to give an explicit expression of the Harish-Chandra C-function of $SU(n, 1)$ for the case when irreducible unitary representations of the maximal compact subgroup $K = S(U(n) \times U(1))$ contain one dimensional unitary representation of $M = S(U(n-1) \times U(1))$. In this paper we compute the matrix element of the Harish-Chandra C-function with respect to the M-highest weight vector of the above M-type. Our main result will give a complete determination of the composition series of the representations which are induced from one dimensional unitary representations of M and characters of the noncompact Cartan subalgebra. Moreover in order to prove the Paley-Wiener type theorem, we need the information on the positions of zeros and poles and their orders of the Harish-Chandra C-function for every K-types which contain certain M-type.

In order to prove our result, using analogous arguments in [3], we obtain the recursion formulae of the Harish-Chandra C-function.

§1. Notation and preliminaries. Let $G = SU(n, 1)$ $(n \geq 2)$ and $K = S(U(n) \times U(1))$. Then K is a maximal compact subgroup of G. Define the analytic subgroups A, N and $\bar{N}$ by

$$A = \left\{ \begin{pmatrix} \cosh t & & \sinh t \\ & I_{n-1} & \\ \sinh t & & \cosh t \end{pmatrix}; t \in \boldsymbol{R} \right\},$$

$$N = \left\{ \begin{pmatrix} 1-\omega/2 & z^* & \omega/2 \\ -z & I_{n-1} & z \\ -\omega/2 & z^* & 1+\omega/2 \end{pmatrix}; z \in \boldsymbol{C}^n, u \in \boldsymbol{R}, \omega = \sum_{i=1}^{n-1} |z_i|^2 + 2\sqrt{-1}u \right\},$$

$$\bar{N} = \left\{ \begin{pmatrix} 1-\omega/2 & z^* & -\omega/2 \\ -z & I_{n-1} & -z \\ \omega/2 & -z^* & 1+\omega/2 \end{pmatrix}; z \in \boldsymbol{C}^n, u \in \boldsymbol{R}, \omega = \sum_{i=1}^{n-1} |z_i|^2 + 2\sqrt{-1}u \right\},$$

where I_{n-1} denotes the unit matrix of order $n-1$ and the asterisk denotes the conjugate transpose. Let M be the centralizer of A in K and $\mathfrak{a}$ be the Lie algebra of A. Then they are given by

*) Faculty of Integrated Arts and Sciences, Hiroshima University.
**) Onomichi Junior College.