6. *On Some Foliations on Ruled Surfaces*

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§0. Introduction. Every ruled surface has a foliation – the ruling –, which characterizes ruled surfaces in all compact complex surfaces. Existence of another foliation characterizes some ruled surfaces in all ruled surfaces. In this paper, we classify ruled surfaces with a foliation on them leaving a curve invariant and having no singularities on it. §1 is a short review of ruled surfaces. The main theorem is stated in §2. To prove it, we need the index formula of Camacho-Sad, which we review in §3. The examples are given in §4. The details of the proof etc. will be found in [10]. The author thanks Prof. T. Suwa for his helpful advices.

§1. A review of ruled surfaces. In this section, we review some properties of ruled surfaces, which may be found in eg. [6].

Definition 1.0. *A ruled surface* $X \xrightarrow{\pi} C$ is a proper holomorphic map of a two-dimensional compact complex manifold X onto a closed Riemann surface C which makes X a $\boldsymbol{P}^1$-bundle over C.

Proposition 1.1. 0) *A ruled surface has a section, i. e. there exists a holomorphic map* $C \xrightarrow{\sigma} X$ *satisfying* $\pi \cdot \sigma = id_C$.

1) *For a ruled surface* $X \xrightarrow{\pi} C$, *there exists a section* C_0 *with the following properties*:

$C_0^{\,2}$ = *the minimum of self-inersection numbers of sections of* $X \xrightarrow{\pi} C$.

We define a number e by

$$e = -C_0^{\,2}, \tag{1.2}$$

which satisfies the following inequality

$$e \geq -g, \tag{1.3}$$

where g is the genus of the Riemann surface C.

For a ruled surface $X \xrightarrow{\pi} C$, the exponential sequences

$$0 \to \mathbf{Z} \to \mathcal{O} \to \mathcal{O}^* \to 0$$

on X and C induce the following commutative diagram of the cohomology long exact sequences.

$$\begin{array}{ccccccccc} \to & \mathrm{H}^1(X, \mathcal{O}_X) & \to & \mathrm{H}^1(X, \mathcal{O}_X^*) & \xrightarrow{c} & \mathrm{H}^2(X, \mathbf{Z}) & \to & \mathrm{H}^2(X, \mathcal{O}_X) \\ & \pi^* \uparrow & & \pi^* \uparrow & & \pi^* \uparrow & & \pi^* \uparrow \\ \to & \mathrm{H}^1(C, \mathcal{O}_C) & \to & \mathrm{H}^1(X, \mathcal{O}_C^*) & \xrightarrow{c} & \mathrm{H}^2(X, \mathbf{Z}) & \to & \mathrm{H}^2(X, \mathcal{O}_X) \end{array}$$

We adopt the following notations:

$$\mathrm{Pic}_0 C = ker[\mathrm{H}^1(C, \mathcal{O}_C^*) \xrightarrow{c} \mathrm{H}^2(C, \mathbf{Z})] \text{ and}$$
$$\mathrm{Pic}_0 X = ker[\mathrm{H}^1(X, \mathcal{O}_X^*) \xrightarrow{c} \mathrm{H}^2(X, \mathbf{Z})].$$

Since