24. *An Example of Elliptic Curve over Q with Rank > 21*

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Abstract: We construct an elliptic curve over Q with rank ≥ 21.

In continuation of our previous paper [1], we give an example of elliptic curve over Q with Q-rank ≥ 21 using Mestre's method. As was explained in [1], any 6-ple $A = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6) \in Z^6$ gives rise to a $Q(T)$-polynomial $r_A(X) = \sum_{i=0}^{5} c_i X^i$, $c_i \in Q(T)$, where $c_5 = 0$ for suitable choices of A. In the following, A will be always chosen so that $c_5 = 0$. Then $Y^2 = r_A(X)$ gives an elliptic curve over $Q(T)$ which will be denoted by $\mathscr{E}_A$. Let $t \in Q$. The elliptic curve over Q obtained from $\mathscr{E}_A$ by specialization $T \to t$ is denoted by $E_{A,t}$. We have defined in [1] the number $S(N, E)$, $S'(N, E)$ for an integer N and an elliptic curve E over Q, and indicated that it is experimentally known that the Q-rank of E is found high when $S(N, E)$, $S'(N, E)$ are large.

Now let $A = (399,380,352,47,4,0)$. (Then we have $c_5 = 0$.) For $\mathscr{E}_A$, $E_{A,t}$ we shall write simply $\mathscr{E}$, E_t. We search in the family of curves

$$\{E_{t_1/t_2} \mid 1 \leq t_1 \leq 20000,\ 1 \leq t_2 \leq 2000,\ t_1\, t_2 \textit{ are co-prime}\},$$

curves satisfying

$S(401) \geq 31.5$, $S'(401) \geq 11.5$, $S(1987) \geq 61$, $S'(1987) \geq 16.5$,
$S(3001) \geq 71$, $S'(3001) \geq 16.5$, $S(4003) \geq 75$, $S'(4003) \geq 16.5$,
$S(5297) \geq 80$, $S'(5297) \geq 17.5$, $S(6581) \geq 84$, and $S'(6581) \geq 20$,

and find $E_{1393/216}, E_{1649/12}, E_{6629/348}, E_{8057/876}$, and $E_{14721/376}$, for the last of which we could show that the Q-rank ≥ 21. Thus we have

Theorem. *Q-rank of $E_{14721/376}$ is ≥ 21.*

In fact, $E_{14721/376}$ is Q-isomorphic to the minimal Weiestrass model

$$y^2 + xy + y = x^3 + x^2 - 215843772422443922015169952702159835x$$
$$- 1947436127778715194725596143545905415150179224132053 5$$

whose conductor is $2*3*5*7*13*17*23*47*4507*$
$115482611374267602141168398241396608699381902319 61$
$\backslash 7225736297616061235976719$.

On this curve the following $P_1, \ldots, P_{21}$ are independent points.

$P_1 = [800843008889340065933/16, 22662214190910903990783584765347/64]$
$P_2 = [1061054106676391459063 7/2209,$
$\quad 1087744114825178454840094794778034/103823]$
$P_3 = [907186946780634143, 728916386168451830641677698]$

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