2. On Foliations on Complex Spaces. II

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§0. Introduction. We defined coherent foliations on reduced complex spaces in the previous papers [5,6]. In this paper, we discuss foliations of dimension one or of codimension one on reduced complex spaces, especially on locally irreducible complex spaces and show that our definition is an extension of the definition by Gómez-Mont [2] of foliations by curves, i.e. the foliation whose leaves are of complex dimension one. Details of proofs are described in [5].

§1. Foliations of dimension one or of codimension one. Let $(X, \mathcal{O}_X)$ be a reduced complex space. We use the following notations:

$\mathcal{M}_X$: the sheaf of germs of meromorphic functions on X
Ω_X: the sheaf of germs of holomorphic 1-forms on X
Θ_X: the sheaf of germs of holomorphic vector fields on X
$\mathrm{sp}X$: the underlying topological space of the complex space X.

For a coherent $\mathcal{O}_X$-module $\mathcal{S}$, we set

$$\mathrm{Sing}\mathcal{S} := \{x \in X \mid \mathcal{S}_x \text{ is not } \mathcal{O}_{X,x}\text{-free}\}.$$

For a coherent $\mathcal{O}_X$-submodule $\mathcal{T}$ of $\mathcal{S}$, we use the notation:

$$S(\mathcal{T}) := \mathrm{Sing}\mathcal{S} \cup \mathrm{Sing}(\mathcal{S}/\mathcal{T}).$$

$S(\mathcal{T})$ is an analytic set in X satisfying $S(\mathcal{T}) \supset \mathrm{Sing}\mathcal{T}$. On $X - S(\mathcal{T})$, $\mathcal{T}$ is locally a direct summand of $\mathcal{S}$.

Our definition of foliations on complex spaces is as follows:

Definition 1.0. Definition a) (by 1-forms).

a.0) A *coherent foliation* on X is a coherent $\mathcal{O}_X$-submodule F of Ω_X satisfying

$$dF_x \subset F_x \wedge \Omega_{X,x} \tag{1.1}$$

at any $x \in X - S(F)$. This condition is called the *integrability condition.* We call $S(F)$ the *singular locus* of the foliation F.

a.1) A coherent foliation $F \subset \Omega_X$ is said to be *reduced* if, for any open subspace $U \subset X$, $\xi \in \Gamma(U, \Omega_X)$ and $\xi|_{U-S(F)} \in \Gamma(U - S(F), F)$ imply $\xi \in \Gamma(U, F)$.

Definition b) (by vector fields).

b.0) A *coherent foliation* on X is a coherent $\mathcal{O}_X$-submodule E of Θ_X satisfying

$$[E_x, E_x] \subset E_x \tag{1.2}$$

at any $x \in X - (S(E) \cup \mathrm{Sing}X)$. This condition is called the *intergrability condition.*

We call $S(E) \cup \mathrm{Sing}X$ the *singular locus* of the foliation E.

b.1) A coherent foliation $E \subset \Theta_X$ is said to be *reduced* if, for any open subspace $U \subset X$, $v \in \Gamma(U, \Theta_X)$ and $v|_{U-(S(E)\cup \mathrm{Sing}X)} \in \Gamma(U - (S(E) \cup \mathrm{Sing}X), E)$ imply $v \in \Gamma(U, E)$.