

47. On Multiplicative Semigroups of Von Neumann Regular Rings

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§1. Introduction. We recall from [1] and [3] that

(1) a monoid M is *left [right] absolutely flat* if any left [right] M -set is flat, and it is *absolutely flat* if it is both left and right absolutely flat, and

(2) a monoid M is *strongly left [right] reversible* if for any $x, y \in M$, there exists $z \in M$ such that $zx = x$ and $zy \in xM \cap yM$ [respectively, $xz = x$ and $yz \in Mx \cap My$], and it is *strongly reversible* if it is both left and right reversible.

Let S be a semigroup and S^1 the monoid obtained by adjoining a new identity 1 if S does not have an identity. Following [1], we say that a semigroup S is *[left, right] absolutely flat* if S^1 is a [left, right] absolutely flat monoid. Similarly, we say that a semigroup S is *[left, right] strongly reversible* if S^1 is a [left, right] strongly reversible.

In [3], Bulman-Fleming and McDowell proved that the multiplicative semigroup of any semi-simple Artinian ring is strongly reversible. This gives an impulse to us for proceeding to our result stated below.

Theorem. *Let R be any ring. Then the following are equivalent:*

- (1) R is a Von Neumann regular ring.
- (2) The multiplicative semigroup of R is strongly reversible.
- (3) The multiplicative semigroup of R is absolutely flat.

Our proof of the theorem is simple and just a combination of a few basic facts concerning idempotents of regular rings.

A semigroup S is called a *semigroup amalgamation base* if for any family $\{T_i \mid i \in I\}$ of oversemigroups T_i of S , there exists a semigroup V in which each T_i is embedded with the property that intersection of every pair of T_i and T_j ($i \neq j$) in V equals S .

It is well-known that absolutely flat semigroups are semigroup amalgamation bases (see [2]).

Here we have

Corollary. *The multiplicative semigroup of any Von Neumann regular ring is a semigroup amalgamation base.*

§2. A proof of theorem. In their paper [2], Bulman-Fleming and McDowell introduced V. Fleischer's characterization of absolutely flat monoids and pointed out that every strongly reversible monoid is absolutely flat. Thus the implication (2) \Rightarrow (3) of Theorem is obtained. It follows from Kilp's theorem [5] (or [1, Proposition 2.5]) that every absolutely flat monoid is regular. Then the implication (3) \Rightarrow (1) is proved. Therefore it suffices to