46. *Some New Examples of Eigenmaps from S^m into S^n*

By Keisuke UENO

Mathematical Institute, Tôhoku University

(Communicated by Heisuke HIRONAKA, M. J. A., June 8, 1993)

1. Introduction. Recently, H. Gauchman and G. Toth introduced a method of constructing new examples of eigenmaps between spheres from known old ones ([1]). As a consequence, they proved the following theorem by applying it to those λ_2-eigenmaps obtained in [2]-[4].

Theorem A. *For $m \geq 5$, there exist full λ_2-eigenmaps*

$$f : S^m \to S^{\frac{m(m+3)}{2}-r},$$

where $r = 1,2,3,4,5,7,11,12,13,16$.

Here a map $f : S^m \to S^n$ is said to be λ_2-eigenmap if all components of f are spherical harmonics of degree 2, and is called full if its image is not contained in any totally hypersphere of S^n.

Theorem A implies, in particular, that full λ_2-eigenmaps $f : S^5 \to S^n$ exist for $n = 4,7,8,9,13,15,16,17,18,19$. However, in their approach the existence of full λ_2-eigenmaps $f : S^5 \to S^n$ is missing for $n = 3,5,6,10,11,12,14$ (for $n = 2$ the non-existence is proved in [1]).

The aim of this note is to show the following theorem which supplements Theorem A.

Theorem B. *Let $k \geq 1$. Then the following hold.*

(i) *There exist full λ_2-eigenmaps $f : S^{2k+1} \to S^l$ for $k^2 + 3k \leq l \leq 2k^2 + 4k + 2$, $l = k^2 + 3k - 2$.*

(ii) *There exist full λ_2-eigenmaps $f : S^{2k+2} \to S^l$ for $k^2 + 5k + 3 \leq l \leq 2k^2 + 6k + 5$, $l = k^2 + 5k - 2 + 2s(k-1)$ $(0 \leq s \leq k+1)$ or $l = k^2 + 5k + 1$.*

Our method of proof is different from that of H. Gauchman and G. Toth and, in fact, makes an essential use of orthogonal multiplications $\boldsymbol{R}^2 \times \boldsymbol{R}^n \to \boldsymbol{R}^r$ in constructing these maps. As a corollary of Theorem B, we obtain for instance

Corollary. There exist full λ_2-eigenmaps $f : S^5 \to S^n$ for $n = 10,11,12,14$.

This corollary combined with a result in H. Gauchman and G. Toth then implies that Theorem A is true for r such that $1 \leq r \leq 13$ or $r = 16$.

2. Existence of orthogonal multiplication for $m = 2$. An orthogonal multiplication $F : \boldsymbol{R}^m \times \boldsymbol{R}^n \to \boldsymbol{R}^r$ is by definition a bilinear map such that $\| F(x, y) \| = \| x \| \cdot \| y \|$, where $\| \cdot \|$ denotes the Euclidean norm. F is said to be full if the image of F spans $\boldsymbol{R}^r$.

It is well-known that if $F : \boldsymbol{R}^n \times \boldsymbol{R}^n \to \boldsymbol{R}^r$ is an orthogonal multiplication, then the Hopf map defined by

$$f_F(x, y) := (\| x \|^2 - \| y \|^2, 2F(x, y)), \quad x, y \in \boldsymbol{R}^n$$