45. *On Integrated Semigroups which are not Exponentially Bounded*

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1. Introduction. Recently, as a generalization of the notion of exponentially bounded n-times integrated semigroups, Hieber [4] introduced that of exponentially bounded α-times integrated semigroups for positive numbers α and obtained interesting results by using Laplace transform techniques. But there exist integrated semigroups which are not exponentially bounded (and do not have the Laplace transforms) (see [5]). It is interesting to study the theory of α-times integrated semigroups which are not necessarily exponentially bounded. In this direction, some results in the special case where α is a nonnegative integer are found in Tanaka and Okazawa [6] and Thieme [7].

In this paper we deal with α-times integrated semigroups which are not necessarily exponentially bounded on a Banach space X for $\alpha \geq 0$. It should be noted that Laplace transform techniques are not available in our case. In §2 we investigate the basic properties of an α-times integrated semigroup and its generator. In §3 we give a characterization of the generator of an α-times integrated semigroup in terms of the associated abstract Cauchy problem. Applying this characterization we prove in §4 the following: (I) (Perturbation Theorem) If A generates an n-times integrated semigroup and if $B \in B(X)$ and $R(B)$ (the range of B) $\subset D(A^n)$ then $A + B$ generates an n-times integrated semigroup. (II) (Adjoint Theorem) If A is the densely defined generator of an α-times integrated semigroup then the adjoint A^* of A generates a β-times integrated semigroup on the adjoint X^* of X for every $\beta > \alpha$. These extend [2, Corollary 3.5] and [4, Corollary 3.7]. The proofs of main results are sketched here, and the details will be published elsewhere.

2. α-times integrated semigroups. Let X be a Banach space with norm $\|\cdot\|$. We denote by $B(X)$ the set of all bounded linear operators from X into itself. Generalizing [1, Definition 3.2] we introduce

Definition 2.1. Let α be a positive number. A family $\{U(t) : t \geq 0\}$ in $B(X)$ is called an *α-times integrated semigroup* on X, if

(a_1) $U(\cdot)x : [0, \infty) \to X$ is continuous for every $x \in X$,

(a_2) $$U(t)U(s)x = \frac{1}{\Gamma(\alpha)}\left(\int_t^{t+s} (t+s-r)^{\alpha-1}U(r)xdr - \int_0^s (t+s-r)^{\alpha-1}U(r)xdr\right)$$

for $x \in X$ and $t, s \geq 0$, where $\Gamma(\cdot)$ denotes the gamma function,

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