44. *Mountain Pass Theorems for Non-differentiable Functions and Applications*

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Abstract: We present some versions of the Mountain Pass Theorem of Ambrosetti and Rabinowitz for locally Lipschitz functionals. A multivalued elliptic problem is solved as an application of these results.

Key words: Clarke subdifferential; critical point theory; multivalued elliptic problem.

1. Introduction. The Mountain Pass Theorem of Ambrosetti and Rabinowitz [1] is a very useful tool for finding critical points of C^1-functionals. We shall give some variants of this celebrated theorem for locally Lipschitz mappings.

Throughout, X will be a real Banach space. As usual, X^* denotes the dual of X and $\langle\cdot,\cdot\rangle$ is the duality pairing between X^* and X. We say that a function $f : X \to \boldsymbol{R}$ is locally Lipschitz ($f \in Lip_{loc}(X, \boldsymbol{R})$) if, for each $x \in X$, there is a neighbourhood V of x and a constant $k = k(V)$ depending on V such that $|f(y) - f(z)| \leq k \| y - z \|$ for each $y, z \in V$.

We recall in what follows the definition of the Clarke subdifferential and some of its most important properties (see, for details, [6]).

For each $x, v \in X$, we define the generalized directional derivative at x in the direction v of a given $f \in Lip_{loc}(X, \boldsymbol{R})$ as

$$f^0(x, v) = \limsup_{y \to x,\, \lambda \searrow 0} (f(y + \lambda v) - f(y)) / \lambda.$$

It is known that, if $f \in Lip_{loc}(X, \boldsymbol{R})$, then $f^0(x, v)$ is a finite number and $|f^0(x, v)| \leq k \| v \|$. The mapping $v \mapsto f^0(x, v)$ is positively homogeneous and subadditive, and then, it is convex continuous. The generalized gradient (the Clarke subdifferential) of f at x is the subset $\partial f(x)$ of X^* defined by $\partial f(x) = \{x^* \in X^* ; f^0(x, v) \geq \langle x^*, v\rangle,\ \forall\, v \in X\}$.

The fundamental properties of the Clarke subdifferential are: a) For each $x \in X$, $\partial f(x)$ is a nonempty convex ★-compact subset of X^*.

b) For each $x, v \in X$, we have $f^0(x, v) = \max\{\langle x^*, v\rangle ; x^* \in \partial f(x)\}$.

c) The set-valued mapping $x \to \partial f(x)$ is upper semi-continuous in the sense that for each $x_0 \in X$, $\varepsilon > 0$, $v \in X$, there is $\delta > 0$ such that for each $x^* \in \partial f(x)$ with $\| x - x_0 \| < \delta$, there exists $x_0^* \in \partial f(x_0)$ such that $|\langle x^* - x_0^*, v\rangle| < \varepsilon$.

d) The function $f^0(\cdot,\cdot)$ is upper semi-continuous.

e) If f attains a local minimum or maximum at x, then $0 \in \partial f(x)$.

f) The function $\lambda(x) = \min\{\| x^* \| ; x^* \in \partial f(x)\}$ exists and is lower

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