6. On Ono's Problem for Quadratic Fields

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For a quadratic number field k, we shall denote by d_k, h_k and χ_k, the discriminant, the class number and the Kronecker character of k, respectively. Let M_k be the Minkowski constant of k:

$$M_k = \begin{cases} \frac{1}{2}\sqrt{d_k} & \text{if } k \text{ is real,} \\ \frac{2}{\pi}\sqrt{-d_k} & \text{if } k \text{ is imaginary.} \end{cases}$$

For the following finite sets of rational prime numbers:

$$S(k) = \{p, \text{ rational prime}; p \leq M_k\},$$
$$S_1(k) = \{p \in S(k)\ ;\ \chi_k(p) = -1\},$$
$$S_2(k) = \{p \in S(k)\ ;\ \chi_k(p) = 0\},$$
$$S_3(k) = \{p \in S(k)\ ;\ \chi_k(p) = 1\},$$

we shall define the following three families of quadratic fields by

$$K_i = \{k, \text{ quadratic field}; S(k) = S_i(k)\}\ (i = 1,2,3).$$

It follows from Minkowski's theorem that the ideal class group of k is generated by the classes of prime ideals $\mathfrak{p}$ lying on p in $S(k)$. Therefore if $S(k) = S_1(k)$ holds, then $h_k = 1$. When k is imaginary, it is easy to prove that $h_k = 1$ holds if and only if $S(k) = S_1(k)$. In the relation with conjecture of Gauss on the class number of real quadratic fields, it is interesting to determine K_1. Leu and Ono determined K_2 and K_3 in [2], [5] as follows:

$$K_2 = \{Q(\sqrt{m})\ ;\ m = -1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 7, 13, 15, \pm 30\},$$
$$K_3 = \{Q(\sqrt{m})\ ;\ m = -1, \pm 2, \pm 3, 5, -7, 13, -15, 17, -23, 33, -47, -71, 73, 97, -119\}.$$

Moreover Leu determined K_1, with one possible exception in [1]:

$$K_1 = \{Q(\sqrt{m})\ ;\ m = -1, \pm 2, \pm 3, 5, -7, -11, 13, -19, 21, 29, -43, 53, -67, 77, -163, 173, 293, 437\}.$$

Remark 1. Under the assumption of GRH (the generalized Riemann Hypothesis), we can determine K_1 without any exception.

Consider the finite set of prime numbers such as

$$S_0(k) = \{p \in S(k),\ \chi_k(p) \neq 1\}.$$

If h_k is odd and $S(k) = S_0(k)$, then $h_k = 1$ holds. The condition that h_k is odd and $S(k) = S_0(k)$ is weaker than $S(k) = S_1(k)$. Our purpose is to determine the family K of all fields k satisfying that h_k is odd and $S(k) = S_0(k)$ under the assumption of GRH.

Theorem 1. *If GRH holds, then there are exactly* 42 *belonging to* K:

$$K = \{Q(\sqrt{m})\ ;\ m = -1, \pm 2, \pm 3, 5, 6, \pm 7, \pm 11, 13, 14, -19, 21, 23, 29, 38, -43, 47, 53, 62, -67, 69, 77, 83, 93, -163,$$