1. *Complete Kähler Manifolds with Zero Ricci Curvature and Kobayashi-Ochiai's Characterization of Complex Projective Spaces*

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In [3] (cf. [9]), Bando and the author proved that there exists a complete Ricci-flat Kähler metric on the complement of a smooth hypersurface D of a Fano manifold X if (X, D) satisfies the conditions: (i) $c_1(X) = \alpha[D]$ with $\alpha > 1$ and (ii) D admits a Kähler-Einstein metric. This result and its proof find some applications in [2], [5], [6] and [11] to problems differential geometry. But we find this existence theorem quite restrictive if we try to apply it to problems in complex algebraic geometry. In this note we announce a general existence theorem for complete Ricci-flat Kähler metrics on certain class of affine algebraic manifolds, which generalizes the results in [3] and [9] by removing the Kähler-Einstein condition at infinity. Details and an application will appear elsewhere ([7]).

Theorem 1. *Let X be a Fano manifold, i.e., X has ample anticanonical bundle. Let D be a smooth connected hypersurface in X such that $c_1(X) = \alpha[D]$ with $\alpha > 1$. Then there exists a complete Ricci-flat Kähler metric on $X - D$.*

The asymptotic bahavior of the resulting Ricci-flat Kähler metric may be described as follows. As $c_1(X) > 0$ and $c_1(X) = \alpha[D]$, there exists a Hermitian metric $\|\cdot\|$ on the line bundle $O_X(D)$ such that

$$\theta = \sqrt{-1}\,\partial\bar{\partial} t \left(t = \log \frac{1}{\|\sigma\|^2}\right)$$

defines a Kähler metric on X, where σ is a holomorphic section of $O_X(D)$ vanishing along D. Then

$$\begin{aligned}\omega &= \sqrt{-1}\,\partial\bar{\partial}\,\frac{n}{\alpha-1}\left(\frac{1}{\|\sigma\|^2}\right)^{\frac{\alpha-1}{n}} \\ &= \left(\frac{1}{\|\sigma\|^2}\right)^{\frac{\alpha-1}{n}}\left(\theta + \frac{\alpha-1}{n}\sqrt{-1}\,\partial t \wedge \bar{\partial} t\right)\end{aligned}$$

turns out to be a complete Kähler metric on $X - D$. The resulting Ricci-flat Kähler metric on $X - D$ has a Kähler potential $\tilde{u}$ of the form

$$\tilde{u} = \frac{n}{\alpha-1}\left(\frac{1}{\|\sigma\|^2}\right)^{\frac{\alpha-1}{n}} + u$$

where u satisfies the *a priori* growth (decay, if $k \geq 3$) estimates:

$$\|\nabla_\omega^k u\| \leq C_k\left\{\left(\frac{1}{\|\sigma\|^2}\right)^{\frac{\alpha-1}{2n}}\right\}^{2-k}$$

for $0 \leq \forall\, k \in Z$, where ∇_ω denotes the Levi-Civita connection of ω. In particular, u is at most of quadratic growth relative to the distance function