22. Pre-special Unit Groups and Ideal Classes of $Q(\zeta_p)^+$

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Let m be a positive integer and $Q(\zeta_m)^+$ the maximal real subfield of the field of m-th roots of unity. Let E_m be the global unit group of $Q(\zeta_m)^+$ and let $\mathcal{C}_m$ be Karl Rubin's special unit group of $Q(\zeta_m)^+$ (see [4]). Then Rubin's main results in [4] implies the following:

Theorem (cf. Th 1.3 and Th 2.2 of [4]). *If* $\alpha: E_m \to Z[\mathrm{Gal}(Q(\zeta_m)^+/Q)]$ *is any* $\mathrm{Gal}(Q(\zeta_m)^+/Q)$*-module map, then* $4\alpha(\mathcal{C}_m)$ *annihilates the ideal class group of* $Q(\zeta_m)^+$.

When m is an odd prime p, our result (Th 3) gives a condition for $\alpha(\mathcal{C}_m)$ to be a "minimal" element that annihilates the ideal class group of $Q(\zeta_p)^+$.

Let p be a fixed prime number and let $\mathcal{S}_p=\{l$; odd prime number such that $l\equiv\pm 1 \pmod p\}$, $\mathcal{S}_p^+=\{l\in\mathcal{S}_p;\ l\equiv 1 \pmod p\}$. For any prime number l in $\mathcal{S}_p$, we denote by $Q(\zeta_p,\zeta_l)^{++}$ the composite field of $Q(\zeta_p)^+$ and $Q(\zeta_l)^+$. We fix any prime ideal $\mathfrak{l}$ of $Q(\zeta_p)^+$ above l and we write $\tilde{\mathfrak{l}}$ for the prime ideal of $Q(\zeta_p,\zeta_l)^{++}$ above $\mathfrak{l}$. Also we fix any generator σ of $G=\mathrm{Gal}(Q(\zeta_p,\zeta_l)^{++}/Q(\zeta_l)^+)$. Let E_p, $E_{p,l}$ be the group of global units of $Q(\zeta_p)^+$, $Q(\zeta_p,\zeta_l)^{++}$ respectively. We define $\mathcal{E}_p(l)=\{\eta\in E_{p,l};\ N_{Q(\zeta_p,\zeta_l)^{++}/Q(\zeta_p)^+}(\eta)=1\}$, $\mathcal{C}_p(l)=\{\varepsilon\in E_p;\ \exists\eta\in\mathcal{E}_p(l)$ such that $\varepsilon^2\equiv\eta \pmod{\prod_{j=0}^{(p-3)/2}\tilde{\mathfrak{l}}^{\sigma^j}}\}$. We call $\mathcal{C}_p(l)$ the *pre-l-special* unit group of $Q(\zeta_p)^+$, and we define the special unit group of $Q(\zeta_p)^+$ by $\mathcal{C}_p=\{\varepsilon\in E_p;\ \varepsilon\in\mathcal{C}_p(l)$ for all but finitely many l in $\mathcal{S}_p\}$ (see [4]).

We fix any sufficiently large integer M, and we put $\mathcal{S}_p^{(M)}=\{l\in\mathcal{S}_p^+;\ l\equiv 1 \pmod{p^M}\}$. Let g_p be a primitive root modulo p such that $\sigma(\zeta_p)=\zeta_p^{g_p}$, and for $i=0,\cdots,(p-3)/2$, let $\varepsilon_i=2/(p-1)\sum_{j=0}^{(p-3)/2}\omega^{-2i}(g_p^j)\sigma^j$ be the idempotents in $Z/p^MZ[G]$, where ω is the Teichmüller character. Then $E_p/E_p^{p^M}=\bigoplus_{i=1}^{(p-3)/2}\varepsilon_i(E_p/E_p^{p^M})$. For each $i=1,\cdots,(p-3)/2$, we take any basis η_i of $\varepsilon_i(E_p/E_p^{p^M})$ and let $\alpha: E_p/E_p^{p^M}\to Z/p^MZ[G]$ be a G-module map such that $\alpha(\eta_i)=\varepsilon_i$. We sometimes use the following condition for l.

Condition-L. *Let l be a prime number in $\mathcal{S}_p^{(M)}$. There is a G-module map*

$$\varphi: (Z[\zeta_p]^+/lZ[\zeta_p]^+)^\times\otimes Z/p^MZ\to Z/p^MZ[G]$$

such that the following diagram is commutative:

$$\begin{array}{ccc} E_p/E_p^{p^M} & \xrightarrow{\ \alpha\ } & Z/p^MZ[G] \\ \downarrow{\scriptstyle\psi} & \nearrow{\scriptstyle\varphi} & \\ (Z[\zeta_p]^+/lZ[\zeta_p]^+)\otimes Z/p^MZ & & \end{array}$$

Here, $Z[\zeta_p]^+$ is the integer ring of $Q(\zeta_p)^+$ and ψ is the reduction map.