19. A Note on Untwisted Deform-spun 2-knots

By Masakazu TERAGAITO
Department of Mathematics, Faculty of Science, Kobe University

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In [5] Litherland introduced the process of deform-spinning of which twist-spinning [8], roll-spinning [1] are particular examples. Given a 1-knot (S^3, K), let g be a self-homeomorphism of (S^3, K) with $g=id$ on a tubular neighbourhood $K\times D^2$ of K. The deform-spun 2-knot corresponding to g is defined as follows.

Fix a point z on K. Take a ball neighbourhood K_- of z in K, and set $B_-=K_-\times D^2$. Let (B_+, K_+) be the complementary ball pair of (B_-, K_-) which is the standard ball pair. Then we construct $\partial(B_+, K_+)\times B^2 \cup_\partial (B_+, K_+)\times_g \partial B^2$, where

$$(B_+, K_+)\times_g \partial B^2=(B_+, K_+)\times I/((x,0)\sim(g(x),1) \text{ for all } x\in B_+).$$

This is a locally-flat sphere pair depending only on the isotopy class γ of g (rel $K\times D^2$). (See [5].) We denote this 2-knot by $(S^4, \gamma K)$, and call it the *deform-spun knot* of K corresponding to γ, or g.

Let $\mathcal{H}(K)$ be the group of self-homeomorphisms g of (S^3, K) with $g=id$ on $K\times D^2$ and let $\mathcal{D}(K)$ be $\mathcal{H}(K)$ modulo isotopy rel $K\times D^2$. We call elements of $\mathcal{D}(K)$ *deformations* of K. It is well-known ([4], [7]) that the exterior $X(K)=\mathrm{cl}(S^3-K\times D^2)$ admits a map $p: X(K)\to\partial D^2$ such that $p|_{\partial X(K)}: \partial X(K)=K\times\partial D^2\to\partial D^2$ is the projection. We will refer to such a map as a *projection* for K. (We always assume that $K\times\theta$ is null-homologous in $X(K)$ for $\theta\in\partial D^2$.) A deformation $\gamma\in\mathcal{D}(K)$ is said to be *untwisted* if there is a projection p for K and a representative g of γ with $p(g|_{X(K)})=p$. If γ is untwisted, we say that γK is untwisted.

For any 1-knot K, twist-spinning $\tau\in\mathcal{D}(K)$ can be defined. (See [5].) Zeeman showed that any ± 1-twist-spun knot $\tau^{\pm 1}K$ of K is unknotted [8]. But the deformation τ is *not* untwisted.

In this note we prove:

Theorem. *There exist infinitely many 1-knots K and untwisted deformations γ of K such that the corresponding untwisted deform-spun 2-knots γK are unknotted.*

Proof of Theorem. For a projection $p: X(K)\to\partial D^2$, if $\theta\in\partial D^2$ is a regular value, then $F^\theta=p^{-1}(\theta)$ is a compact, codimension 1 submanifold of $X(K)$ and $\partial F^\theta=K\times\{\theta\}$. That is, F^θ is a *Seifert surface* for K. (See [4], [7].) Let $\gamma\in\mathcal{D}(K)$ be an untwisted deformation and let g be a representative of γ with $p(g|_{X(K)})=p$. Then $g(F^\theta)=F^\theta$ for each $\theta\in\partial D^2$. A tubular neighbourhood of γK is $\partial K_+\times D^2\times B^2\cup K_+\times D^2\times\partial B^2$ and so γK has the exterior