17. *Inverse Iteration Method with a Complex Parameter*

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1. **Introduction.** The inverse iteration method has been used as one of the most powerful ways for computing eigenvectors. And the theory of error estimates has been established almost completely as is seen in Wilkinson [2]. Let A be a symmetric (n, n) matrix and let $\{\lambda_k, \phi_k\}$, $k=1, \cdots, n$, be pairs of eigenvalues and the corresponding eigenvectors of A. The inverse iterarion process for the eigenvector ϕ_j is to solve the following linear equations with initial data $z^{(0)}$ under the conditions $|\lambda_j-\lambda| \ll c < |\lambda_k-\lambda|$, $(k \neq j)$:

$$(1.1) \qquad (A-\lambda I)z^{(m+1)}=z^{(m)}, \quad m=0,1,2,\cdots.$$

In this paper, we propose to introduce into this method a new technique, which is simple but effective in practical computations. Our method is to solve the same linear equation, but with a complex parameter $\lambda+\sqrt{-1}\tau$ instead of real λ in (1.1) and to carry out the next iteration process after substituting the imaginary part of the solution for the initial vector. We can show that the imaginary part y of the solution of the linear equation contains the component of the aimed eigenvector far more than the real part x. The ratio of the l^2 norms $\|x\|/\|y\|$ can be used to derive a sharp error estimate for the computed eigenvector. It may be emphasized that the error bound given by (2.6) in Theorem 2.2 is rather effective so that one can judge how many digits in actual computations are correct in significant decimals by estimating the right hand side of (2.6). It is also emphasized that in our method the efficiency of enriching the component of the aimed eigenvector is almost doubled compared with the standard traditional method.

In §2, we explain our method and state the theorems. In §3, we show some propositions which describe how our method works. When we refer to the traditional method based on (1.1), we call it, for brevity, the standard method. Our main purpose here is to present the idea of our method. So throughout this paper, we state our theory as if rounding errors were zero.

2. **A new method with a complex parameter and the theorems.** Let A be a real (n, n) matrix which is symmetric and has n different eigenvalues. Let $\{\lambda_k, \phi_k\}$, $k=1,2,3,\cdots,n$, be pairs of eigenvalues and the corresponding normalized real eigenvectors of A. First we describe our method for computing the eigenvector ϕ_j corresponding to the eigenvalue λ_j under the following assumption.