13. *Levi Conditions for Hyperbolic Operators with a Stratified Multiple Variety*

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1. Introduction and result. Let $P(x,D_x)$ be a differential operator of order m, i.e. $P(x,D_x)=P_m(x,D_x)+P_{m-1}(x,D_x)+\cdots$, where $P_j(x,D_x)$, $j=0,\cdots,m$, denotes the homogeneous part of order j of P (here $D_x=(1/i)\partial/\partial x$). We assume that P has C^∞ (smooth) coefficients in the open subset $\Omega\subset\boldsymbol{R}^{n+1}$, and that $0\in\Omega$. Consider the principal symbol of P, $p_m(x,\xi)$, which we shall assume to be a homogeneous polynomial of degree m with real valued smooth coefficients; we say that P is hyperbolic with respect to the direction ξ_0 if the equation $p_m(x,\xi)=0$, where $x=(x_0,x_1,\cdots,x_n)$, $\xi=(\xi_0,\xi_1,\cdots,\xi_n)$, has only real roots in ξ_0. It has long been well known that if P is strictly hyperbolic, i.e. if all the above mentioned roots of $p_m(x,\xi)=0$ are distinct, then the Cauchy problem

$$P(x,D_x)=f,\qquad \partial_0^j u|_{x_0=0}=g_j,\qquad j=0,\cdots,m-1,$$

is well posed. Well posedness, roughly speaking, means that there exists a unique distribution solution for any choice of the distributions f and g_j's. On the other hand, if the roots of $p_m(x,\xi)$ are not distinct, it is well known that in general we have well posedness only if we assume some conditions on the lower order terms, see e.g. [7] and [9] in the case of double roots, [10] and [11] in the case of roots of higher multiplicity.

When roots of higher multiplicity occur an important object is the localised principal symbol: If $d^j p_m(\rho)=0$, $j=0,\cdots,r-1$, and $d^r p_m(\rho)\neq 0$, define $p_{m,\rho}(\delta z)=\lim_{t\to 0}t^{-r}p_m(\rho+t\delta z)$, where $\delta z\in T_\rho(T^*\Omega)$, the tangent space at ρ of $T^*\Omega\simeq\Omega\times\boldsymbol{R}_\xi^{n+1}$.

In this note we present a result on necessary conditions for the well posedness of the Cauchy problem for P. Here is a list of the assumptions we make:

(H_1) The principal symbol $p_m(x,\xi)$ is real and hyperbolic with respect to ξ_0.

(H_2) The characteristic roots of $\xi_0\mapsto p_m(x,\xi_0,\xi')$ have multiplicity of order at most 3 and $\mathrm{Char}\,P=\{(x,\xi)\,|\,p_m(x,\xi)=0\}=\Sigma_1\cup\Sigma_2\cup\Sigma_3$, where

$$\Sigma_1=\{(x,\xi)\in T^*\Omega\,|\,p_m(x,\xi)=0,\ dp_m(x,\xi)\neq 0\},$$
$$\Sigma_2=\{(x,\xi)\in T^*\Omega\,|\,p_m(x,\xi)=0,\ dp_m(x,\xi)=0,\ d^2p_m(x,\xi)\neq 0\},$$
$$\Sigma_3=\{(x,\xi)\in T^*\Omega\,|\,p_m(x,\xi)=0,\ dp_m(x,\xi)=0,\ d^2p_m(x,\xi)=0\}.$$

Here and in the sequel $x'=(x_1,\cdots,x_n)$ and analogously for ξ'.

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