10. The Aitken-Steffensen Formula for Systems of Nonlinear Equations. V

By Tatsuo NODA
Faculty of Engineering, Toyama Prefectural University

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1. Introduction. Let $x=(x_1, x_2, \cdots, x_n)$ be a vector in R^n and D a region contained in R^n. Let $f_i(x)$ $(1\leqq i\leqq n)$ be real-valued nonlinear functions defined on D and $f(x)=(f_1(x), f_2(x), \cdots, f_n(x))$ an n-dimensional vector-valued function. Then we shall consider a system of nonlinear equations

(1.1) $$x=f(x),$$

whose solution is $\bar{x}$.

As mentioned in [2]–[4], Henrici [1, p. 116] has considered a formula, which is called the Aitken-Steffensen formula. Now, we have studied the above Aitken-Steffensen formula for systems of nonlinear equations in [2]–[4], and shown [2, Theorem 2], [3, Theorem 2] and [4, Theorem 1].

The purpose of this paper is to show Theorem 4 by combining [2, Theorem 2] with [2, Theorem 1], and Theorem 5 by using only the relation in [4, Theorem 1].

2. Statement of results. For any $x\in R^n$ and an $n\times n$ matrix $A=(a_{ij})$, we shall use the norms $\|x\|$ and $\|A\|$ defined by

$$\|x\|=\max_{1\leqq i\leqq n}|x_i| \quad \text{and} \quad \|A\|=\max_{1\leqq i\leqq n}\sum_{j=1}^{n}|a_{ij}|,$$

respectively. Let $U(\bar{x})=\{x\,;\,\|x-\bar{x}\|<\delta\}\subset D$ be a neighbourhood.

Given $x^{(0)}\in R^n$, define $x^{(i)}\in R^n$ $(i=1,2,\cdots)$ by

(2.1) $$x^{(i+1)}=f(x^{(i)}) \quad (i=0,1,2,\cdots).$$

Put

(2.2) $$d^{(i)}=x^{(i)}-\bar{x} \qquad \text{for } i=0,1,2,\cdots,$$

and then define an $n\times n$ matrix D_k by

$$D_k=(d^{(k)}, d^{(k+1)}, \cdots, d^{(k+n-1)}).$$

Throughout this paper, we shall assume the same conditions (A.1)–(A.5) as in [2].

(A.1) $f_i(x)$ $(1\leqq i\leqq n)$ are two times continuously differentiable on D.

(A.2) There exists a point $\bar{x}\in D$ satisfying (1.1).

(A.3) $\|J(\bar{x})\|<1$, where $J(x)=(\partial f_i(x)/\partial x_j)$ $(1\leqq i, j\leqq n)$.

(A.4) The vectors $d^{(k)}, d^{(k+1)}, \cdots, d^{(k+n-1)}$, $k=0, 1, 2, \cdots$, are linearly independent.

(A.5) $\inf\{|\det D_k|/\|d^{(k)}\|^n\}>0$.

Then, we shall consider the Aitken-Steffensen formula

(2.3) $$y^{(k)}=x^{(k)}-\Delta X^{(k)}(\Delta^2X^{(k)})^{-1}\Delta x^{(k)} \quad (k=0,1,2,\cdots),$$