84. Remarks to our Former Paper, "Uniform Distribution of Some Special Sequences"

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Abstract: In [2], Y. H. Too pointed out that our proof of Theorem 2 of our former paper [1] contained an error. In this paper, we shall first restate the main results of [1], [2] as Theorems A, B, C, then give a revised proof of Theorem B (= Theorem 2 [1]), prove a Proposition which, combined with Theorem A (= Theorem 1 [1] which was correctly proved), yields Theorem C and finally remark that Theorem B can also be easily deduced from Theorem A.

Let p_n be the n-th prime number.

Theorem A (Theorem 1 [1]). *Let $f(x)$ be a continuously differentiable function with $f(x) \to \infty$ $(x \to \infty)$. If $f'(x)\log x$ is monotone, $n\,|f'(n)| \to \infty$ as $n \to \infty$, and*

$$f(n)/(\log n)^l \to 0 \quad (n \to \infty) \text{ for some } l > 1,$$

then $(\alpha f(p_n))$ is uniformly distributed mod 1, *where $\alpha (\neq 0)$ is any real constant.*

Theorem B (Theorem 2 [1]). *Let $f(x)$ be a continuously differentiable function with $f'(t) > 0$ and $f''(t) > 0$. If $t^2 f''(t) \to \infty$ as $t \to \infty$ and*

$$f(n)/(\log n)^l \to 0 \quad (n \to \infty) \quad \text{for some } l > 1,$$

then $(\alpha f(p_n))$ is uniformly distributed mod 1, *where $\alpha (\neq 0)$ is any real constant.*

Theorem C (Theorem 3 [2]). *Let f be a twice differentiable function with $f \to \infty$, $f' > 0$ and $f'' < 0$. If $x^2(-f''(x)) \to \infty$, $(\log x)^2(-f''(x))$ is decreasing as $x \to \infty$ and $f(n)/(\log n)^l \to 0 (n \to \infty)$ for some $l > 1$, then $(\alpha f(p_n))_1^\infty$ is uniformly distributed* mod 1, *where $\alpha(\neq 0)$ is any real constant.*

Revised proof of Theorem B. The proof becomes correct if we change the estimation of I_2 in [1 : p.84 line 6 ↑ through p.85 line 3] as follows:

We choose any sequence $c_N \to \infty$ as $N \to \infty$, and put

$$I_2 = \int_2^{p_N} \frac{e^{2\pi i h f(t)}}{\log t}\,dt = \left(\int_2^{c_N} + \int_{c_N}^{p_N}\right) \frac{e^{2\pi i h f(t)}}{\log t}\,dt = A + B, \text{ say.}$$

Then clearly

$$|A| = \left| \int_2^{c_N} \frac{e^{2\pi i h f(t)}}{\log t}\,dt \right| \le \int_2^{c_N} \frac{dt}{\log t} \ll \frac{c_N}{\log c_N}.$$

Now applying [3 : Lemma 10.2], we get

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