80. *Primitive π-regular Semigroups*

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Abstract: In this note we investigate the structure of π-regular semigroups, the nonzero idempotents of which are primitive.

Various characterizations for primitive regular semigroups have been obtained by T. E. Hall [4], G. Lallement and M. Petrich [6], G. B. Preston [7], O. Steinfeld [8] and P. S. Venkatesan [9], [10](this appeared also in the book of A. H. Clifford and G. B. Preston [3]). J. Fountain [5] considered primitive abundant semigroups. In this paper we consider primitive π-regular semigroups and in this way we generalize the previous results for primitive regular semigroups.

Throughout this paper, $\boldsymbol{Z}^+$ will denote the set of all positive integers. If S is a semigroup with zero 0, we will write $S = S^0$ and $S^* = S - \{0\}$.

An element a of a semigroup $S = S^0$ is a *nilpotent* if there exists $n \in \boldsymbol{Z}^+$ such that $a^n = 0$. The set of all nilpotents of a semigoup S is denoted by $Nil(S)$. A semigroup S is a *nil-semigroup* if $S = Nil(S)$. An ideal I of a semigroup $S = S^0$ is a *nil-ideal* of S if I is a nil-semigroup. An ideal extension S of a semigroup K is a *nil-extension* of K if S/K is a nil-semigroup. By $R^*(S)$ we denote *Clifford's radical* of a semigroup $S = S^0$, i.e. the union of all nil-ideals of S (it is the greatest nil-ideal of S).

A semigroup S is *π-regular* (*completely π-regular*) if for every $a \in S$ there exist $n \in \boldsymbol{Z}^+$ and $x \in S$ such that $a^n = a^n x a^n$ ($a^n = a^n x a^n$ and $a^n x = x a^n$). A semigroup S is *π-inverse* if S is *π-regular* and every regular element of S has a unique inverse. If A is a nonempty subset of a semigroup S, then by $\mathrm{Reg}(A)$ $(E(A))$ we denote the set of all regular elements (idempotents) of A. If e is an idempotent of a semigroup S then we denote by G_e the maximal subgroup of S with e as its identity. A nonzero idempotent e of a semigroup $S = S^0$ is *primitive* if for every $f \in E(S^*)$, $f = ef = fe \Rightarrow f = e$, i.e. if e is minimal in $E(S^*)$, relative to the partial order on $E(S^*)$. A semigroup $S = S^0$ is *primitive* if all of its nonzero idempotents are primitive.

For undefined notion and notations we refer to [2] and [3].

Lemma 1. *Let $S = S^0$ be a semigroup. If eS (Se) is a 0-minimal right (left) ideal of S generated by a nonzero idempotent e, then e is primitive.*

Proof. For a proof see Lemma 6.38 [3].

The converse of the previous lemma is not true. For example, in the semigroup $S = \langle a, e, 0 \mid a^2 = 0, e^2 = e, ae = 0, ea = a, a0 = 0a = e0 =$

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