9. The Structure of Compactifications of C^3

By Mikio FURUSHIMA

Department of Mathematics, College of Education, University of the Ryukyu

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Introduction. Let (X, Y) be a smooth projective compactification of C^3 with the second Betti number $b_2(X)=1$. Then Y is an irreducible ample divisor on X with $\operatorname{Pic} X \cong Z\mathcal{O}_X(Y)$ and the canonical divisor K_X can be written as $K_X \sim -rY$ $(r>0, r \in Z)$ (cf. [1]). Thus X is a Fano threefold of first kind (cf. [6]). The integer r is called the index of X.

Two smooth compactifications (X, Y) and (X', Y') are said to be isomorphic, denoted by $(X, Y) \cong (X', Y')$, if there is a biregular morphism $\alpha : X \to X'$ such that $\alpha(Y) = Y'$.

Then we have:

Theorem. (1) $r \geq 4 \Rightarrow (X, Y) \cong (P^3, P^2)$, *in fact*, $r=4$;

(2) $r=3 \Rightarrow (X, Y) \cong (Q^3, Q_0^2)$,

(3) $r=2 \Rightarrow (X, Y) \cong (V_5, H_5^0)$ *or* (V_5, H_5^∞),

(4) $r=1 \Rightarrow (X, Y) \cong (V_{22}, H_{22}^0)$ *or* (V_{22}, H_{22}^∞).

Remark 1. (1) (P^3, P^2), (Q^3, Q_0^2), (V_5, H_5^0), (V_5, H_5^∞) are determined uniquely up to isomorphism (cf. [5], [8]).

(2) (V_{22}, H_{22}^0), (V_{22}, H_{22}^∞) are not unique, in fact, they have a 4-dimensional family ([7]).

Notation. Q^3: a smooth quadric hypersurface in P^4

Q_0^2: a quadric cone in P^3

V_5: a linear section $\operatorname{Gr}(2,5) \cap P^6$ of the Grassmann $\operatorname{Gr}(2,5) \hookrightarrow P^9$ (Plücker embedding) by three hyperplanes in P^9, which is the Fano threefold of the index two, degree 5 in P^5

H_5^0: a normal hyperplane section of V_5 with exactly one rational double point of A_4-type, which is also the degenerated del-Pezzo surface of degree 5

H_5^∞: a non-normal hyperplane section of V_5 whose singular locus is a line Σ with the normal bundle $N_{\Sigma|V_5} \cong \mathcal{O}_\Sigma(-1) \oplus \mathcal{O}_\Sigma(1)$, in particular, H_5^∞ is a ruled surface swept out by lines in V_5 intersecting the line Σ

V_{22}: the Fano threefold of index one with the genus $g=12$, degree 22 in P^{13} (the anti-canonical embedding)

H_{22}^0 (resp. H_{22}^∞): a non-normal hyperplane section of V_{22} whose singular locus is a line Z with the normal bundle $N_{Z|V_{22}} \cong \mathcal{O}_Z(-2) \oplus \mathcal{O}_Z(1)$, and the multiplicity $\operatorname{mult}_Z H_{22}^0$ (resp. $\operatorname{mult}_Z H_{22}^\infty$) of H_{22}^0 (resp. H_{22}^∞) at a general point of Z is equal to two (resp. three), in particular, H_{22}^∞ is a ruled surface swept out by conics in V_{22} intersecting the line Z.

The proof of Theorem in the case of $r \geq 2$ was given in [2], [5], [8].