77. *Spectral Concentration and Resonances for Unitary Operators*

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1. Introduction. Operator-theoretical approach to the theory of resonances for a family of selfadjoint operators H_k has been investigated by J. S. Howland ([1]), A. Orth ([4]) and W. Hunziker ([2]). (For other works see the references in these papers, and [3; VIII, §5].) In particular, Orth established a link between the theory of resonances and the limiting absorption principle, developed the theory without any analyticity assumptions, and applied it successfully to N-body Schrödinger operators using the Mourre estimate.

In the present note we are mainly interested in the abstract part of the work [4] and shall present a generalization which can cover H_k given by a form sum. (Note that in [4] it is supposed that $H_k \supset H_0 + \kappa W$.) To this end we find it convenient to construct a counterpart of Orth's abstract results for a unitary operator family U_k. It will be given in §2. In §3 we transform the results to the selfadjoint families. This amounts to considering the Cayley transform $(H_\kappa - i)(H_\kappa + i)^{-1}$ of H_κ, or $(H_\kappa - d)^{-1}$ if H_κ is uniformly semibounded. In §4 we apply the results to a simple example in which a Dirichlet decoupled ordinary differential operator is perturbed by a delta type measure.

In this note we present only results. Detailed proofs will be published elsewhere ([5]).

The main instrument in [1] and [4] is the Livsic matrix. It is generally defined as follows.

Definition (L). Let T be a densely defined closed operator in a Hilbert space $\mathbf{H}$ and P be a finite dimensional orthogonal projection. Then the Livsic matrix $B(T, z)$ of T in $P\mathbf{H}$ is a finite dimensional operator defined by

$$P(T-z)^{-1}P = (B(z, T) - z)^{-1},$$

where z belongs to the resolvent set $\rho(T)$ of T.

2. Spectral concentration for unitary operators. Let U be a unitary operator and P be an orthogonal projection onto the m-dimensional space $K = P\mathbf{H}(m < \infty)$. It is not necessary that U and P commute. We put $\Omega_0 := \{w \in C\,;\, |w| > 1\}$. We shall consider the Livsic matrix $B(w)$ of U in K. For $w \in \Omega_0$ $B(w)$ is well-defined and given as

$$B(w) = PUP - PU\overline{P}(\overline{U} - w)^{-1}\overline{P}UP$$

where $\overline{U} = \overline{P}U\overline{P}$.

Let $U_\kappa = \int_0^{2\pi} e^{i\theta}\, dF_\kappa(\theta)$, $\kappa \geq 0$, be unitary operators such that $U_\kappa \to U_0$ in the strong sense as $\kappa \to 0$. And let $e^{i\theta_0}$ be an eigenvalue of U_0