6. *The Determination of the Imaginary Abelian Number Fields with Class Number One*

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Uchida proved that there exist only finitely many imaginary abelian number fields with class number one [12], and gave the value 2×10^{10} as an upper bound of the conductors of such fields [13]. Several authors determined such fields of some types, but not all of them have been determined yet. (Masley determined the cyclotomic number fields with class number one [8], and Uchida determined such fields of two power degrees [14].)[1] The purpose of this article is to report that we have determined all the imaginary abelian number fields with class number one in proving the following. (The details will appear elsewhere [15].)

Theorem. *There exist exactly* 171 *imaginary abelian number fields with class number one as given in the attached table. Among them,* 29 *fields are cyclotomic,* 49 *fields are cyclic, and* 87 *fields are maximal with respect to inclusion. The maximal conductor of these fields is* $10921 = 67\cdot 163$, *which is the conductor of the biquadratic number field* $\boldsymbol{Q}(\sqrt{-67}, \sqrt{-163})$.

Now we sketch here the method of proof. The basic idea is due to Uchida. (See [13] [14].) In the following, let K be an imaginary abelian number fields with class number $h(K)$. When $h(K)=1$, the genus number[2] of K is one, which is equivalent to say that the character group X corresponding to K is a direct product of subgroups generated by a character of prime power conductor:

$$X=\langle\chi_1\rangle\times\cdots\times\langle\chi_r\rangle, \text{ the conductor } f_{\chi_i} \text{ of } \chi_i = \text{a prime power.}$$

Moreover, when $h(K)=1$, the subfield of K corresponding to $\langle\chi_i\rangle$ has strict class number one ([13], Prop. 1). Therefore, we need to consider only K of this type. Among K of this type, we first determine K with $h^-(K)=1$, and then check whether $h^+(K)=1$, or not. Here, $h^-(K)$ (resp. $h^+(K)$) denotes the relative class number of K (resp. the class number of the maximal real subfield of K). $(h(K)=h^-(K)h^+(K).)$ The key idea which facilitates the determination is:

($*$) If $h(K)=1$, then, for any subfield F of K such that K/F is totally ramified at a finite prime, the strict class number of F is one.

From this, in most cases, we can immediately restrict K to be considered

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[1] Recently, Louboutin determined imaginary abelian sextic number fields with class number one [4]. The author knew his result after completion of the present article.

[2] The extension degree $[K^*, K]$, where K^* is the genus field of K.