1. On the Poincaré-Bogovski Lemma on Differential Forms

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1. Introduction. Integrability conditions for differential forms go back to Poincaré [10, Section II]. Let D be a bounded domain in $\boldsymbol{R}^n$. What is called the Poincaré lemma (cf. [11, Theorem 4.11]) asserts that every smooth closed differential form on D is exact provided that D is starshaped. This is proved by constructing a (linear) integral operator I such that

(1) $$d(I\omega)+I(d\omega)=\omega,$$

where ω is a form on D and d denotes the exterior derivative. Indeed, $d\omega=0$ implies that ω has a potential $I\omega$. However, for usual choice of I, found for example in [11, Theorem 4-11], the support of $I\omega$, $\operatorname{spt} I\omega$, may not be compact in D even if ω is compactly supported in D.

Our goal in this paper is to construct an integral operator K satisfying (1) with $I=K$ such that $\operatorname{spt} K\omega$ is compact if $\operatorname{spt}\omega$ is compact. (More precisely we will show that $\operatorname{spt} K\omega \subset D\cup\Gamma$ if $\operatorname{spt}\omega\subset D\cup\Gamma$ where Γ is an open subset on ∂D.) We also prove that K is bounded in L^p Sobolev spaces.

Bogovski [1], [2] first constructed such K on n-forms ω satisfying $\int_D \omega=0$ (even for an arbitrary bounded Lipschitz domain D); in this case d equals the divergence operator. As noticed in [1, Theorem 4] such a property on $K\omega$ is important for localizing a closed form by preserving closedness. His operator K is applied to various analyses on incompressible viscous fluid (cf. [3], [4], [6], [7], [9], [12], [13]).

Borchers and Sohr [5] and Griesinger [8] treated such a problem on the operator rot. In fact Griesinger [8] constructed an integral operator on a bounded domain D starshaped with respect to a ball in D although she didn't prove (1).

In this paper we extend Bogovski's formula for the exterior derivatives on a bounded domain starshaped with respect to a ball.

2. Formula of potentials. We first give an explicit formula of K. Let $D\subset\boldsymbol{R}^n$ be a bounded domain starshaped with respect to a closed ball B in D, i.e., $D=\{tx+(1-t)y \mid x\in D,\ y\in B,\ t\in[0,1]\}$. Let B' be a closed ball in the interior of B. For $k=1,\cdots,n$ and given $h\in C^\infty(B)$ satisfying $\operatorname{spt} h \subset B'$ and $\int_{B'} h\,dx=1$, we set

$$H_k(x,y)=\int_1^\infty h(y+t(x-y))t^{k-1}(t-1)^{n-k}dt.$$

Let $\mathcal{D}^k$ denote the space of C^∞ k-forms compactly supported in D. For