79. A Class of Inclusion Theorems Associated with Some Fractional Integral Operators†)

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(Communicated by Shokichi IYANAGA, M. J. A., Nov. 12, 1991)

In the present paper the authors prove several inclusion theorems for some interesting subclasses of analytic functions involving a certain family of fractional integral operators. The corresponding results for the Hardy space $\mathcal{H}^p$ $(0<p\leqq\infty)$ follow as corollaries of these theorems. Some applications to the generalized hypergeometric functions are also considered.

1. Introduction. Let $\mathcal{A}$ denote the class of functions $f(z)$ *normalized* by

$$f(z)=z+\sum_{n=2}^{\infty}a_n z^n, \tag{1.1}$$

which are analytic in the *open* unit disk

$$\mathcal{U}=\{z : |z|<1\}.$$

Definition 1. A function $f(z)\in\mathcal{A}$ is said to be in the class $\mathcal{R}(\gamma)$ if it satisfies the inequality:

$$\mathrm{Re}\{f'(z)\}>\gamma \quad (z\in\mathcal{U};\ 0\leqq\gamma<1).$$

The class $\mathcal{R}(0)$ was studied systematically by MacGregor [6] who indeed referred to numerous earlier investigations involving functions whose derivative has a positive real part. Various interesting subclasses of $\mathcal{A}$ associated with the class $\mathcal{R}(\gamma)$ were considered elsewhere by (among others) Sarangi and Uralegaddi [11], Owa and Uralegaddi [8], and Srivastava and Owa [12].

Let $\mathcal{T}$ be the subclass of $\mathcal{A}$ consisting of functions of the form:

$$f(z)=z-\sum_{n=2}^{\infty}|a_n|z^n, \tag{1.2}$$

and denote by $\mathcal{R}^*(\gamma)$ the class obtained by taking the intersection of the classes $\mathcal{R}(\gamma)$ and $\mathcal{T}$; that is,

$$\mathcal{R}^*(\gamma)=\mathcal{R}(\gamma)\cap\mathcal{T} \quad (0\leqq\gamma<1). \tag{1.3}$$

Finally, let $\mathcal{H}^p$ $(0<p\leqq\infty)$ denote the Hardy space of analytic functions $f(z)$ in $\mathcal{U}$, and define the integral means $M_p(r, f)$ by

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†) 1991 Mathematics Subject Classification. Primary 30C45, 26A33; Secondary 33C20, 30H05.

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