73. A Note on Exponents of K-groups of Rings of Algebraic Integers

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1. In this note, we construct higher K-groups of rings of algebraic integers with arbitrary large l-exponent using the technique developed by K. Komatsu in his papers [4] [5].

Let l be an odd prime number. For an algebraic number field F, by which we always mean an algebraic extension over the field of rational numbers $\boldsymbol{Q}$ of finite degree, we denote by $\mathcal{O}_F$ the ring of algebraic integers of F, by F_∞ the cyclotomic $\boldsymbol{Z}_l$-extension of F, by F_m its m-th layer i.e., the unique cyclic extension of F contained in F_∞ of degree l^m. For an abelian torsion group X and a positive integer n, define $X_n=\{x\in X \mid l^n x=0\}$ and $X_\infty=\bigcup_{n=1}^\infty X_n$. We also define the *l-exponent* of the group X to be $\exp(X)=\max\{l^n \mid X_n\neq 0\}$. Let μ be the group of roots of unity. And we choose a generator ζ_n of each μ_n with $\zeta_{n+1}^l=\zeta_n$. For each odd integer ν, let $K_{2\nu}(\mathcal{O}_F)$ be the Quillen's 2ν-th K-group. According to Quillen [6], $K_{2\nu}(\mathcal{O}_F)$ is an abelian group of finite order.

Let k be a totally real algebraic number field. For a while, we fix a non-negative integer n_0 and put

$$k^{(n_0)}=k\cdot\boldsymbol{Q}_{n_0-1},\quad K^{(n_0)}=k^{(n_0)}(\mu_1),\quad G_\infty^{(n_0)}=\mathrm{Gal}(K_\infty^{(n_0)}/k^{(n_0)}),$$
$$\Gamma^{(n_0)}=\mathrm{Gal}(K_\infty^{(n_0)}/K^{(n_0)}),\quad\text{and}\quad \Delta^{(n_0)}=\mathrm{Gal}(K_\infty^{(n_0)}/k_\infty^{(n_0)}).$$

Let $\chi\colon \Delta^{(n_0)}\to\boldsymbol{Z}_l^\times$ be the Teichmüller character i.e., a homomorphism such that $\zeta_1^\delta=\zeta_1^{\chi(\delta)}$ for all $\delta\in\Delta^{(n_0)}$ and

$$\varepsilon_i=(\#(\Delta^{(n_0)}))^{-1}\sum_{\delta\in\Delta^{(n_0)}}\chi(\delta)^i\delta^{-1}\in\boldsymbol{Z}_l[\Delta^{(n_0)}]$$

the canonical orthogonal idempotent for each integer i. We choose a topological generator γ of $\Gamma^{(n_0)}$ and define an l-adic integer κ by $\zeta_m^\gamma=\zeta_m^\kappa$ $(m\geq 1)$. Let $\mathcal{T}=\varprojlim_k \mu_k$ be the Tate module, which is a free $\boldsymbol{Z}_l$-module of rank 1 and on which $G_\infty^{(n_0)}$ acts in a natural way. If X is a $G_\infty^{(n_0)}$-module, which is also a $\boldsymbol{Z}_l$-module, we define, for each integer $n\geq 0$,

$$X(n)=X\otimes_{\boldsymbol{Z}_l}\mathcal{T}\otimes_{\boldsymbol{Z}_l}\cdots\otimes_{\boldsymbol{Z}_l}\mathcal{T}\quad (n\text{ times}),$$

endowed with diagonal action of $G_\infty^{(n_0)}$. We denote, as usual, by $X^{G_\infty^{(n_0)}}$ the $G_\infty^{(n_0)}$-invariant submodule of X.

We shall prove a preliminary lemma.

Lemma 1. *Let X be an l-primary $G_\infty^{(n_0)}$-module and n a non-negative*

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