68. *Generalized Interface Evolution with the Neumann Boundary Condition*

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(Communicated by Kunihiko KODAIRA, M. J. A., Oct. 14, 1991)

1. Introduction. We are concerned with geometric evolution (e.g. motion by mean curvature) of interfaces in a smoothly bounded domain Ω $(\subset \boldsymbol{R}^n)$ whose boundary $\partial\Omega$ perpendicularly intersects with interfaces. In [12] the second author extended a level set approach introduced by Chen-Giga-Goto [1] and Evans-Spruck [3] to this type of the Neumann problem and obtained a unique global weak solutions for the initial value problem provided that Ω is convex. This note reports that the convexity assumption of Ω can be removed. The details and proofs will appear elsewhere.

One of key ingredients is the comparison principle for the Neumann boundary value problem for singular degenerate parabolic equations. For the Neumann problem this principle is first established by Lions [10] for the Hamilton-Jacobi equations. For nonsingular degenerate elliptic equations the comparison principle is established by Ishii and Lions [9]. See also [8] for more general oblique boundary conditions. However, their argument does not apply to singular equations. In [12] the second author obtained the comparison principle for our problem assuming that Ω is convex. His method appeals to the idea of [6] by regarding $\partial\Omega$ as space infinity. Unfortunately, the choice of test functions does not apply to general domains. In this note we construct test functions by using local coordinate patches near $\partial\Omega$ so that they apply to general domains.

In [7] Huisken considers the interface intersecting perpendicularly with $\partial\Omega$ and moving by mean curvature. He constructed a global smooth evolution of interfaces when Ω is a cylindrical domain $D\times\boldsymbol{R}$ and the initial interface is the graph of a smooth function on D, where D is bounded. Although our theory presented below assumes that Ω is bounded, it can be extended to cylindrical domain $D\times\boldsymbol{R}$ provided D is bounded. The motion by mean curvature with right contact angle at $\partial\Omega$ arises as a singular limit of a reaction-diffusion equation with the Neumann condition [11].

2. Comparison principle. We here present a simple and typical version of our comparison principle rather than stating its general form to avoid technical complexity. We consider an evolution equation of the form

$$u_t+F(\nabla u, \nabla^2 u)=0 \quad \text{in } Q=(0,T)\times\Omega \tag{1}$$

$$\partial u/\partial\nu=0 \quad \text{on } S=(0,T)\times\partial\Omega, \tag{2}$$