66. *Selberg Zeta Functions and Ruelle Operators for Function Fields*

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§ 1. **Introduction.** Nowadays Selberg zeta functions are defined for semi-simple Lie groups G of real rank one and its discrete subgroups Γ, since Selberg defined it first in [11] in 1956. Two ways are known for studying Selberg zeta functions. One is by Selberg trace formulas and the other is by Ruelle operators. By the former method, Selberg zeta functions are finally expressed as the determinants of the Laplacians ([10], [2], [7], [6]). By the latter method, they are expressed as the determinants of **1**-(Ruelle operator) ([9], [3], [8]). Above all, the result of Mayer [8] is remarkable in number theoretic viewpoint. He treats the Selberg zeta function of $\Gamma = PSL(2, \boldsymbol{Z})$, which is the unique example of non-compact $\Gamma \backslash G$ for which the second method is applied.

In this paper we fulfill the second method in the case $\Gamma = PGL(2, \boldsymbol{F}[T])$, where $\boldsymbol{F}$ is a finite field of order q of odd characteristic. As is well be seen in number theory, Γ has similar properties to those of $PSL(2, \boldsymbol{Z})$. We can apply the second method by following Mayer [8]. For the present Γ, Akagawa [1] constructs Selberg trace formulas and proves that Selberg zeta functions are rational with respect to q^s. This paper will give another proof of Akagawa's result. It is much shorter than the original one, as is seen in the case of $PSL(2, \boldsymbol{Z})$ by Mayer [8]. In the next section, we will define continued fractions in function fields and deduce some properties. In the third section, we will classify conjugacy classes of Γ following Akagawa [1]. In the last section, we will define Ruelle operators on function spaces over function fields, and establish the relation to Selberg zeta functions.

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§ 2. **Continued fractions.** Throughout this paper, we fix q a power of an odd prime and $\boldsymbol{F}$ the finite field of order q. We denote R, K, and F to be $\boldsymbol{F}[T]$, $\boldsymbol{F}(T)$, and $\boldsymbol{F}((T^{-1}))$, which are analogues of $\boldsymbol{Z}$, $\boldsymbol{Q}$, and $\boldsymbol{R}$. The field F is the completion of K with respect to the place T^{-1}. Elements in F are expressed as $x = \sum_{i=-\infty}^{k} a_i T^i$ ($a_i \in \boldsymbol{F}$, $k \in \boldsymbol{Z}$, $a_k \neq 0$). We will write deg for the map from F to $\boldsymbol{Z}$ such that the element x corresponds to k. The map deg