61. A Note on Poincaré Sums of Galois Representations. II

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Let k be a field of characteristic zero, K a finite Galois extension of k and χ the character of a k-representation ρ of the Galois group $G=G(K/k)$. The subfield corresponding to Ker ρ is written K_χ because Ker $\rho=$ Ker $\chi^* \stackrel{\text{def}}{=} \{s \in G\,;\, \chi^*(s)=1\}$ where we set $\chi^*(s)=\chi(s)/\chi(1)$. In [5], we proved

(0.1) $$K_\chi = k(P_\chi)$$

where

(0.2) $$P_\chi = \sum_{s \in G} \theta^s \chi(s) \qquad \text{(a Poincaré sum)}$$

and θ is a normal basis element for K/k, chosen once for all.

If, in particular, K/k is a cyclic Kummer extension of degree n with $G=\langle s \rangle$, $\rho(s)=\zeta$, this being a primitive nth root of 1 in k, then $K=k(P_\chi)$ as well as $P_\chi^n \in k$, a property peculiar to this K/k. Usually P_χ is referred to as the Lagrange resolvent and satisfies

(0.3) $$P_\chi^s = \chi(s^{-1}) P_\chi.$$

Therefore, it is natural to seek a generalization of (0.3) for any Galois extension K/k such that G splits over k[1].

In this paper, we shall prove among others that

(0.4) $$P_\chi^{a(s)} = \chi^*(s^{-1}) P_\chi, \quad s \in G, \quad \chi \in \mathrm{Irr}\,(G)^{2)}$$

where

(0.5) $$a(s) = \frac{1}{n} \sum_{t \in G} tst^{-1}, \qquad n=[K:k]=|G|.$$

This $a(s)$ is an element of the center $k[G]_0$ of the group ring $k[G]$ and is viewed as an endomorphism of the vector space K over k. When k is a number field, (0.4) implies that

(0.6) $$P_\chi^{\alpha_{K/k}(\mathfrak{p})} = \chi^*\left(\left[\frac{K/k}{\mathfrak{P}}\right]^{-1}\right) P_\chi, \qquad \mathfrak{P} \mid \mathfrak{p},$$

where $\alpha_{K/k}$ is the generalized Artin map introduced and studied in the series of papers [2], [3], [4].

1. Operator $a(s)$. Let K/k be a finite Galois extension of fields of characteristic zero with $G=G(K/k)$. Fix once for all a normal basis element θ for K/k. Assume that k is a splitting field for G. We begin with a description of the following diagram

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1) By a theorem of Brauer ([1], p. 86, (16.3)), this is always the case if k contains a primitipe mth root of 1 where m is the exponent of G.

2) Irr (G) denotes the set of all absolutely irreducible characters of G.