55. *Tuboids of C^n with Cone Property and Domains of Holomorphy*

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Abstract: Let X be a C^∞-manifold, M a closed submanifold, Ω an open set of M. We introduce in §1 a class of domains U of X called Ω-tuboids. They coincide with the original ones by [2] apart from an additional assumption, of cone type, at $\partial\Omega$. In §2 we take a complex of sheaves $\mathcal{F}$ on X and denote by $\mu_\Omega(\mathcal{F})$ the microlocalization of $\mathcal{F}$ along Ω. We take a closed convex proper cone λ of T^*_MX and describe the stalk of $\boldsymbol{R}\pi_*\boldsymbol{R}\Gamma_\lambda\mu_\Omega(\mathcal{F})_{T^*_MX}$ by means of cohomology groups of $\mathcal{F}$ over Ω-tuboids U with profile $\gamma=\operatorname{int}\lambda^{oa}$. In §3 we take $X=\boldsymbol{C}^n$, $M=\boldsymbol{R}^n$, Ω open convex in M and prove that in the class of Ω-tuboids with a prescribed profile there is a fundamental system of domains of holomorphy. By this tool we prove in §4 a decomposition theorem for the microsupport at the boundary SS_Ω by Schapira [9] (cf. also [5]).

§1. Let X be a C^∞ manifold, M a closed submanifold, let $\tau: TX\to X$ (resp $\pi: T^*X\to X$) be the tangent (resp cotangent) bundle to X, and let $\tau: T_MX\to M$ (resp $\pi: T^*_MX\to M$) be the normal (resp conormal) bundle to M in X. We note that we have an embedding $\iota: TM\hookrightarrow M\times_X TX$ and a projection $\sigma: M\times_X TX\to T_MX$. For a subset A of X (resp of M) we shall define the strict normal cone of A in X (resp M) by $N^X(A)=TX\setminus C(X\setminus A, A)$ (resp $N^M(A)=TM\setminus C(M\setminus A, A)$) where $C(\cdot,\cdot)$ is the closed cone of TX defined in [6]. If no confusion may arise, we shall omit the superscripts X and M. Let Ω be an open set of M and x_0 a point of $\partial\Omega$. We shall assume

$$N^M_{x_0}(\Omega)\neq\emptyset. \tag{1.1}$$

Let γ be an open convex cone of $\bar\Omega\times_M T_MX$ with $\tau(\gamma)\supset\bar\Omega$.

Definition 1.1. A domain $U\subset X$ is said to be an Ω-tuboid with profile γ when

$$\sigma(M\times_X TX\setminus C(X\setminus U, \bar\Omega))\supset\gamma. \tag{1.2}$$

One proves that $\theta\in T_{x_0}X\setminus C_{x_0}(X\setminus U, \bar\Omega)$ iff for a choice of local coordinates there exists a neighborhood V of x_0 and an open cone G containing θ s.t. $((\bar\Omega\cap V)+G)\cap V\subset U$. In particular:

$$TX\setminus C(X\setminus U, \bar\Omega)=(TX\setminus C(X\setminus U, \bar\Omega))+N(\Omega).$$

Lemma 1.2. *Let* (1.2) *hold. Then there exists an open convex cone* $\beta\subset\bar\Omega\times_X TX$:

$$\beta\subset TX\setminus C(X\setminus U, \bar\Omega),\quad \beta=\beta+N(\Omega),\quad \sigma(\beta)\supset\gamma. \tag{1.3}$$

Proof. For a choice of coordinates on X we identify

$$M\times_X TX\cong TM\oplus_M T_MX\ni(t, x+\sqrt{-1}\,y). \tag{1.4}$$