48. An Ill-posed Estimate for a Class of Degenerate Quasilinear Elliptic Equations

By Kazuya HAYASIDA
Department of Mathematics, Faculty of Science, Kanazawa University

(Communicated by Kunihiko KODAIRA, M. J. A., June 11, 1991)

§1. **Introduction.** Let D be a domain in R^N, and let Γ be an open subset of ∂D, which is said to be an initial surface. We denote by O an origin in R^N. We suppose that O is the interior point of Γ. Let L be an elliptic operator in $\bar{D}$, which may be nonlinear. Let u be a solution of $L(u)=0$ in D. Then the ill-posed estimate in Cauchy's problem is the following: There are an open neighborhood U of O and two constants C, δ with $0<\delta<1$ such that

$$\|U\|_{2,U\cap D}\leqq C(\|u\|_{1,\Gamma})^{\delta}(\|u\|_{3,D})^{1-\delta}, \tag{1.1}$$

where $\|\ \|_i$ $(i=1,2,3)$ are some norms on Γ, $U\cap D$ and D, respectively. In particular, $\|\ \|_{1,\Gamma}$ means some quantity of initial data of u. The investigation with respect to ill-posed estimates of linear operators is referred to John's work [2]. The Hadamard's three circles theorem is close to the estimate (1.1). With respect to the nonlinear case, Výborný [7] has proved recently the Hadamard's three circles theorem for nonlinear uniformly elliptic operators.

The estimate (1.1) implies immediately the unique continuation property, which asserts that $u=0$ in $U\cap D$ if the initial data of u vanishes on Γ. For elliptic operators with linear principal parts the unique continuation property was extensively studied by many authors. Let $A(x,\xi)$ be a mapping from $D\times R^N$ into R^N such that for a.e. $x\in R^N$ and for all $\xi\in R^N$

$$|A(x,\xi)|\leqq C|\xi|^{p-1},\qquad A(x,\xi)\cdot\xi\geqq c|\xi|^p$$

where c, $C>0$ and $p>1$. Then we consider particularly the elliptic operator L with

$$L(u)=\operatorname{div}(A(x,\nabla u)\cdot\nabla u). \tag{1.2}$$

Recently, Martio [5] gave a counterexample of the form (1.2) such that the unique continuation property does not hold. In his counterexample, the function $A(x,\xi)$ and $u(x)$ are constructed skillfully under the conditions such as $p=N\geqq 3$, $D=\{x_N>0\}$ and $\Gamma=\{x_N=0\}$.

When $N=2$, the unique continuation property holds for the operators of (1.1) under some conditions (see e.g., [1] and [4]). However these method can not be applied to the case of $N\geqq 3$. The difficulty is originated from the degeneration of ellipticity. Thus there arises a question: If $N\geqq 3$, does the unique continuation property, moreover the ill-posed estimate hold for degenerate quasilinear elliptic operators?

In this paper we give a partial affirmative answer for the above ques-