68. *The Aitken-Steffensen Formula for Systems of Nonlinear Equations. IV*

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1. Introduction. Let $x=(x_1, x_2, \cdots, x_n)$ be a vector in R^n and D a region contained in R^n. Let $f(x)$ be a real-valued nonlinear function defined on D. We denote by $R^{n\times n}$ the set of all $n\times n$ real matrices. Define an n-dimensional vector $\nabla f(x)$ and an $n\times n$ matrix $H(x)$ by

$$\nabla f(x)=(\partial f(x)/\partial x_i) \qquad (1\leqq i\leqq n)$$

and

$$H(x)=(\partial^2 f(x)/\partial x_j \partial x_k) \qquad (1\leqq j, k\leqq n).$$

For any $x\in R^n$, we shall use the norms $\|x\|$ and $\|x\|_2$ defined by

$$\|x\|=\max_{1\leqq i\leqq n}|x_i| \quad \text{and} \quad \|x\|_2=\left(\sum_{i=1}^n x_i^2\right)^{1/2},$$

respectively. The corresponding matrix norms, denoted by $\|A\|$ and $\|A\|_s$, are defined as

$$\|A\|=\max_{1\leqq i\leqq n}\sum_{j=1}^n |a_{ij}| \quad \text{and} \quad \|A\|_s=\lambda^{1/2},$$

respectively, where $A=(a_{ij})\in R^{n\times n}$, and λ is the maximum eigenvalue of A^*A, A^* being the transposed matrix of A. We also define the matrix norm $\|A\|_E$ by

$$\|A\|_E=\left(\sum_{i=1}^n\sum_{j=1}^n a_{ij}^2\right)^{1/2}.$$

In this section, we shall assume the same conditions (A.1)–(A.4) as in [5] except for (A.1).

(A.1) $f(x)$ is three times continuously differentiable on D.

(A.2) There exists a point $\bar{x}\in D$ satisfying $\nabla f(x)=0$.

(A.3) The $n\times n$ symmetric matrix $H(\bar{x})$ is positive definite.

(A.4) β is a constant satisfying $0<\beta<2$.

We see that $f(x)$ has a local minimum at $\bar{x}$ by conditions (A.1)–(A.3). For computational purpose, we have proposed in [5, (2.1)] an iteration method

$$x^{(k+1)}=x^{(k)}-\frac{\beta}{\|H(x^{(k)})\|_E}\nabla f(x^{(k)}) \tag{1.1}$$

for finding $\bar{x}$ under conditions (A.1)–(A.4).

As mentioned in [2], [3] and [4], Henrici [1, p. 116] has considered a formula, which is called the Aitken-Steffensen formula. Now, we have studied the above Aitken-Steffensen formula for systems of nonlinear equations in [2], [3] and [4], and shown [2, Theorem 2], [3, Theorem 2] and [4, Theorem 1].