66. Comparison Principle for Singular Degenerate Elliptic Equations on Unbounded Domains*)

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1. Introduction. This paper, as a preliminary study for [5], announces a comparison principle for viscosity solutions of singular degenerate elliptic equations

(1) $u+F(x, u, \nabla u, \nabla^2 u)=0$ in Ω ($\nabla u=\mathrm{grad}\, u$, $\nabla^2 u$: Hessian)

where Ω is a domain (not necessarily bounded) in $\boldsymbol{R}^n$. A typical example is

$$\lambda u-|\nabla u| \operatorname{div}\left(\frac{\nabla u}{|\nabla u|}\right)=0 \quad (\lambda \in \boldsymbol{R}). \tag{2}$$

This equation is derived from the mean curvature flow equation

$$v_t-|\nabla v| \operatorname{div}\left(\frac{\nabla v}{|\nabla v|}\right)=0 \tag{2'}$$

by setting $v(t, x)=e^{\lambda t}u(x)$; see also [1, 4]. The idea of the proof applies to parabolic equation in [5], so we omit the detailed proof since it is easily seen from the argument in [5]. The comparison principle for viscosity solutions is established by M.G. Crandall and P.L. Lions [3] for first order equations, by P.L. Lions [12], R. Jensen [10], H. Ishii [7] for second order degenerate elliptic equations (see also [11]), by Y.-G. Chen, Y. Giga and S. Goto [1] for singular parabolic equations including the mean curvature flow equations (see also [4]). However so far no results applied for (2) in an unbounded domain.

2. Comparison principle. Let Ω be a domain in $\boldsymbol{R}^n$ not necessarily bounded. We consider a degenerate elliptic equation of form

(3) $u+F(x, u, \nabla u, \nabla^2 u)=0$ in Ω.

In this paper we call a continuous function $m\colon [0, \infty)\to[0, \infty)$ a *modulus* if $m(0)=0$ and it is nondecreasing. We first list assumptions on $F=F(x, r, p, X)$.

(F1) $F\colon J(\Omega)=\Omega\times\boldsymbol{R}\times(\boldsymbol{R}^n\backslash\{0\})\times S^n\to\boldsymbol{R}$ is continuous, where S^n denotes the space of real $n\times n$ symmetric matrices.

(F2) F is *degenerate elliptic*, i.e., $F(x, r, p, X+Y)\leq F(x, r, p, X)$ in $J(\Omega)$ if $Y\geq O$.

(F3) $-\infty<F_*(x, r, 0, O)=F^*(x, r, 0, O)<\infty$ for all $(x, r)\in\Omega\times\boldsymbol{R}$, where F_* and F^* are, respectively, *the lower and upper semicontinuous relaxation* (*envelope*) of F on $J(\bar{\Omega})$, i.e.,

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