48. *Twisting Symmetry-spins of Pretzel Knots**)

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Let π be the commutator subgroup of the knot group of a knot in the 4-sphere S^4. In [1] it is shown that if π is finite, then $\pi = G \times Z_d$ where $G=\{1\}$, the quaternion group $Q(8)$, the binary icosahedral group I^* or the generalized binary tetrahedral group $T(k)$ and d is an odd integer which is relatively prime to the order of G. Conversely, Yoshikawa [10] has shown that these groups can be realized as the commutator subgroups of the knot groups of knots in S^4 except $Q(8)\times Z_d$, $d>1$. Actually, these knots were constructed by twist-spinning certain 2-bridge knots and pretzel knots. The exceptional groups were realized only as the commutator subgroups of knot groups of knots in homotopy 4-spheres. Note that $Q(8)\times Z_d$ is isomorphic to the fundamental group of a prism manifold M_d, that is, the Seifert fibered manifold with invariants $\{b : (o_1, 0) : (2,1), (2,1), (2,1)\}$, $d = |2b+3|$ (cf. [3], [7]). Since then, by using the deform-spinning introduced by Litherland [6], Kanenobu [4] and the author [9] showed that for $d=5$, 11, 13 and 19 (equivalently $b=-4, 4, -8$ and 8), there is a fibered 2-knot in S^4 whose fiber is the punctured prism manifold M_d°; thus for these values of d, the groups $Q(8)\times Z_d$ are realized as the commutator subgroups of knot groups of knots in S^4. It should be noted that a fibered 2-knot with fiber $M_d^\circ (d>1)$ cannot be constructed by twist-spins (cf. [2]).

The purpose of this paper is to show that other three values can be realized.

Theorem. *There exists a fibered 2-knot in S^4 whose fiber is a punctured prism manifold M_d° with fundamental group isomorphic to $Q(8)\times Z_d$ for $d=$ 3, 5, 11, 13, 19, 21, 27.*

Our examples for the cases $d=3$, 21, 27 will be constructed by a product of two symmetry-spinnings and 1-twist-spinning for pretzel knots. It is unknown whether there exists such a fibered 2-knot in S^4 for any other value of d.

All maps and spaces are assumed to be in the PL category, and all manifolds are oriented. A circle is identified with the quotient space $\boldsymbol{R}/\boldsymbol{Z}$. The unit interval $[0, 1]$ is denoted by I.

1. Construction. Let (S^3, K) be a knot and suppose that there are orientation-preserving periodic homeomorphisms $g_i (i=1, 2)$ on (S^3, K) of order n_i such that $g_1 g_2 = g_2 g_1$, $(n_1, n_2)=1$, and $J_1 \cup J_2$ is the Hopf link with $lk(J_1, J_2)=1$, where $J_i = \mathrm{Fix}(g_i)$, $(i=1, 2)$. Let $n=n_1 n_2$, $g=g_1 g_2$. Let q:

*) Dedicated to Professor Junzo TAO on his 60th birthday.