47. Symmetries of the Garnier System and of the Associated Polynomial Hamiltonian System

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Introduction. The aim of this note is to present a group of symmetries for the Garnier system, which is a system of partial differential equations obtained by monodromy preserving deformations of second order Fuchsian differential equations on $\boldsymbol{P}^1$, and for the associated polynomial Hamiltonian system.

Let us consider a Pfaffian system:

$$E(\theta): \qquad dx_i=\sum_{j=1}^{n} G_{ij}(x,t,\theta)dt_j \qquad i=1,\cdots,m,$$

where $G_{ij}(x,t,\theta)$ are rational functions in $(x,t)=(x_1,\cdots,x_m,t_1,\cdots,t_n)$ depending on parameters $\theta\in\boldsymbol{C}^N$ and d stands for the exterior differentiation with respect to (x,t). For a birational transformation $S\colon(x,t)\to(x',t')$, we denote by $S\cdot E(\theta)$ the system of differential equations in the variables (x',t') obtained from $E(\theta)$ by the transformation S.

Definition. A *group of symmetries* for the system E is a group whose element is a pair $\sigma=(S,l)$ of a birational transformation $S\colon(x,t)\to(x',t')$ and an affine transformation $l\colon\boldsymbol{C}^N\to\boldsymbol{C}^N$ such that $S\cdot E(\theta)=E(l(\theta))$.

Let $\sigma=(S,l)$ and $\sigma'=(S',l')$ be symmetries of the system E. The product $\sigma\cdot\sigma'$ and the inverse σ^{-1} to σ are defined by $\sigma\cdot\sigma':=(S\circ S',l\circ l')$ and $\sigma^{-1}:=(S^{-1},l^{-1})$, respectively.

1. Garnier system $\mathcal{G}_n$ and the associated system $\mathcal{H}_n$. The n-dimensional Garnier system is the Hamiltonian system

$$\mathcal{G}_n: \qquad d\lambda_i=\sum_{j=1}^{n}\{K_j,\lambda_i\}dt_j, \qquad d\mu_i=\sum_{j=1}^{n}\{K_j,\mu_i\}dt_j,$$

$i=1,\cdots,n$, where $\{\cdot,\cdot\}$ stands for the Poisson bracket

$$\{f,g\}=\sum_i\left(\frac{\partial f}{\partial\mu_i}\frac{\partial g}{\partial\lambda_i}-\frac{\partial g}{\partial\mu_i}\frac{\partial f}{\partial\lambda_i}\right).$$

The Hamiltonians $K_i=K_i(\theta,\lambda,\mu,t)$ are given by

$$K_i=M_i\sum_{k=1}^{n}M^{k,i}\left\{\mu_k^2-\sum_{m=1}^{n+2}\frac{\theta_m-\delta_{im}}{\lambda_k-t_m}\mu_k+\frac{\kappa}{\lambda_k(\lambda_k-1)}\right\},$$

where $\theta_1,\cdots,\theta_{n+2}$, $\kappa:=(1/4)(\sum_{i=1}^{n+2}\theta_i-1)^2-(1/4)\theta_\infty^2$ are constants, $t_{n+1}=0$, $t_{n+2}=1$, and

$$M_i=-\frac{\Lambda(t_i)}{T'(t_i)}, \qquad M^{k,i}=\frac{T(\lambda_k)}{(\lambda_k-t_i)\Lambda'(\lambda_k)},$$

$(i,k=1,\cdots,n,n+1,n+2)$ defined by using